Equivalence of the approaches to QFT at finite temperature in Minkowski spacetime

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Outline

1. The algebraic approach to perturbative QFT (pAQFT)
2. Thermofield dynamics (TFD)
3. The contour approach of Schwinger and Keldysh
4. Conclusion

The algebraic approach to perturbative QFT (pAQFT)

- Interplay between fields of
  - algebraic approach (esp. QFT on CST)
  - deformation quantization
  - path integral approach to perturbative QFT
- Here: M Minkowski spacetime, Field content: one real scalar field
- Starting point: Space of all configurations (resp. space of all histories)

$$\mathcal{E} = C^\infty(M) \quad \longleftrightarrow \quad \text{off-shell formalism}$$

- Observables are smooth functionals of the configurations $C^\infty(\mathcal{E}) \ni A$:

$$\frac{\delta^n}{\delta\phi^n} A(\phi) \equiv A^{(n)}(\phi) \in \mathcal{E}'(M^n)$$

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Algebra of local observables

Classical Algebra $\mathcal{A}_{\text{cl}}$

The space $\mathcal{F}$ of smooth, compactly supported functionals $A \in C^\infty(\mathcal{E})$, such that

$$\mathrm{WF}\,\frac{\delta^n}{\delta\phi^n}A(\phi) \subset \left\{(x_1,\ldots,x_n,k_1,\ldots,k_n) \in \dot{T}M^n : \sum_{i=1}^n k_i = 0\right\}$$

constitutes a commutative algebra with the pointwise product $(A\cdot B)(\phi) = A(\phi)B(\phi)$ which is called $\mathcal{A}_{\text{cl}} = (\mathcal{F},\cdot)$.

- can be equipped with a Poisson bracket $\{\cdot,\cdot\}$
- fulfills $\{A,B\} = 0$ if $\operatorname{supp} A$ spacelike separated from $\operatorname{supp} B$

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$$A \star B = A \cdot B + \sum_{n=1}^{\infty} \frac{\hbar^n}{n!} \Gamma^n (A \otimes B)$$

$$\Gamma = \int \mathrm{dx}\ \mathrm{dy}\ H(x,y) \frac{\delta}{\delta\phi(x)} \otimes \frac{\delta}{\delta\phi(y)} \quad \longleftrightarrow \quad \text{gen. Wick's theorem}$$

H is a bi-distribution of Hadamard-form

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Perturbation theory

Interaction

Let V be an local interaction functional of the form

$$V(\phi) = \frac{1}{n} \int \mathrm{d}x \; g(x)\phi(x)^n, \qquad g \in \mathcal{D}(M), \qquad g = 1 \text{ on } \mathcal{O} \subset M$$

inducing interacting field equations $P\phi + \phi^{n-1} = 0$ on $\mathcal{O}$.

Retarded operators

We construct a linear map R_V on $\mathcal{A}$ with the properties

- $R_V \left(P\Phi_f + \Phi_f^{n-1} \right) = P\Phi_f$ on $\mathcal{O}$
- $R_{V_1+V_2}(A) = R_{V_1}(A)$ if $\mathrm{supp}(V_2)$ is later than $\mathrm{supp}(A)$

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Causal perturbation theory

Define inductively (in $\hbar$) a time-ordered product for local functionals

$$A \cdot_{\mathcal{T}} B = \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \Gamma_F^n (A \otimes B) = \begin{cases} A \star B & \text{supp}(A) \text{ later than } \text{supp}(B) \\ B \star A & \text{supp}(B) \text{ later than } \text{supp}(A) \end{cases}$$

where in Γ_F, H is replaced by $H_F = H + i\Delta_A$.
Up to order $\hbar^n$ this amounts to extending the distributions[a]

$$\mathcal{D}'(M^k \setminus \{0\}) \ni \mathring{H}_F^k \to H_F^k \in \mathcal{D}'(M^k), \qquad k = 1, \dots, n$$

[a] The extension is ambiguous $\leftrightarrow$ renormalization freedom

Then one can define the S-matrix $\mathcal{S}(V) = e_{\cdot_{\mathcal{T}}}^{iV}$ (up to $\hbar^n$) and:

$$R_V(A) = \mathcal{S}(V)^{\star -1} \star (\mathcal{S}(V) \cdot_{\mathcal{T}} A)$$

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Interacting theory

The algebra $\mathcal{A}_V(\mathcal{O})$, which is generated by the $R_V(A)$ with $A \in \mathcal{A}(\mathcal{O})$ is called the interacting algebra of observables. Its algebraic structure does not depend on the choice of g outside of $\mathcal{O}$ and we obtain a net

$$\mathcal{O} \longrightarrow \mathcal{A}_V(\mathcal{O})$$

of interacting algebras and $\mathcal{A}_V = \lim_{\mathcal{O} \nearrow M} \mathcal{A}_V(\mathcal{O})$ adiabatic limit.

Generating functional for interacting time-ordered products

Let A_f, $f \in \mathcal{D}(\mathcal{O})$ be a local field. The generating functional of the expectation values of interacting time-ordered products of A_f in a state ω is given by

$$Z(f) = \omega(\mathcal{S}(V)^{\star -1} \star \mathcal{S}(V + A_f))$$

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Motivation to extend the formalism

Vacuum sector

Let ω be the vacuum state of the free theory. Then ω translates to a state on $\mathcal{A}_V(\mathcal{O})$, which exists in the adiabatic limit and fulfills

$$Z(f) \xrightarrow{\text{a.l.}} \frac{\omega(\mathcal{S}(V + A_f))}{\omega(\mathcal{S}(V))}$$

in the sense of generating functionals: Gell-Mann and Low formula.

Finite temperature

Let ω_β be the KMS-state of the free theory.

- ω_β translates to some state on $\mathcal{A}_V(\mathcal{O})$: adiabatic limit: unknown
- similar factorization not expected since, spectrum condition was crucial $\rightarrow$ Ansatz: modify theory

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Thermofield dynamics (TFD)

Main idea

Let $\mathfrak{M}$ be the von-Neumann algebra associated to the scalar field ϕ_f with $f \in \mathcal{D}$ in the KMS-state ω_β and

- $j(A) = JAJ$ the modular conjugation, obtained by Tomita-Takesaki theory
- $\mathfrak{B}$ the *-algebra generated by $\mathfrak{M}$ and $j(\mathfrak{M}) \cong \mathfrak{M}'$

For an (extended) interaction $\hat{V} = V - j(V) \in \mathfrak{B}$ we assume to get

$$Z(f) \xrightarrow{\text{a.l.}} \omega_\beta(\mathcal{S}_{\mathfrak{B}}(\hat{V} + A_f)) = Z_{\mathcal{B}}(f)$$

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Realization in the pAQFT approach

Configuration space

Enlarge the field content: $\hat{\mathcal{E}} = \mathcal{E} \oplus \mathcal{E}$.

Enlarged algebra $\mathcal{B}$

Define a $\star$-product on functionals in $\mathcal{F}(\hat{\mathcal{E}})[[\hbar]]$

$$\mathbf{\Delta}_+(x,y) = \begin{pmatrix} D_+^\beta(x,y) & D_+^\beta(x,y+i\beta/2) \\ D_+^\beta(x,y+i\beta/2) & D_+^\beta(y,x) \end{pmatrix}, \quad \beta = \frac{e^0}{k_B T}$$

where D_+^β is the KMS two-point functions of the scalar field.

Subalgebras $\mathcal{B}_1$ and $\mathcal{B}_2$

$$\mathcal{B}_1 = \{A \in \mathcal{B} : \frac{\delta}{\delta\phi_2} A = 0\}, \quad \mathcal{B}_2 = \{A \in \mathcal{B} : \frac{\delta}{\delta\phi_1} A = 0\}$$

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$$\mathcal{B}_1 = \{A \in \mathcal{B} : \frac{\delta}{\delta\phi_2} A = 0\}, \quad \mathcal{B}_2 = \{A \in \mathcal{B} : \frac{\delta}{\delta\phi_1} A = 0\}$$

Generalized modular conjugation

Generalized modular conjugation

We define a map

$$j : \mathcal{B} \to \mathcal{B}, \qquad (jA)(\phi, \psi) = A^*(\psi, \phi), \quad j = j^{-1}$$

Properties of $\mathcal{B}$

- $\mathcal{B}_1 \cong \mathcal{A}$
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Perturbation theory

Time-ordered product

We introduce a time-ordered product $\cdot_{\mathcal{T}}$ by a matrix-valued Feynman propagator

$$\mathbf{\Delta}_F(x,y) = \begin{pmatrix} D_F^\beta(x,y) & D_+^\beta(x,y+i\beta/2) \\ D_+^\beta(x,y+i\beta/2) & D_{aF}^\beta(x,y) \end{pmatrix}$$

where $D_{(a)F}^\beta$ is the (anti-) Feynman propagator for the KMS state.

S-Matrix

The S-Matrix $\mathcal{S}_{\mathcal{B}}(\hat{V})$ of the extended theory is given by the time-ordered exponential[a]

$$\mathcal{S}_{\mathcal{B}}(\hat{V}) = \exp_{\cdot_{\mathcal{T}}}(i\hat{V}), \qquad \hat{V} = V - j(V), \quad V \in \mathcal{B}_1$$

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Equivalence

Interacting observables

The map $R_{\hat{V}}$ corresponding to $S_{\mathcal{B}}(\hat{V})$ in $\mathcal{B}$ is given by

$$R_{\hat{V}}(A) = \mathcal{S}_{\mathcal{B}}(\hat{V})^{\star -1} \star \left[\mathcal{S}_{\mathcal{B}}(\hat{V}) \cdot_{\mathcal{T}} A\right]$$
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Conclusion, Adiabatic limit

Adiabatic limit

Since the algebras generated by R_V and $R_{\hat{V}}$ acting on $\mathcal{A}(\mathcal{O})$ coincide: This approach does not help on deciding, whether ω_β translates to a KMS-state on $\mathcal{A}_V$.

Back to motivation

If, ω_β exists in the adiabatic limit and

- $\mathcal{S}(V)\,|\Omega_\beta\rangle$ tends to a translation invariant vector
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on the interacting Hilbert space, then

$$Z_{\mathcal{B}}(f) = \omega_\beta(\mathcal{S}_{\mathcal{B}}(\hat{V} + A_f)) = \langle \Omega_\beta |\, j(\mathcal{S}(V))\mathcal{S}(V + A_f)\, |\Omega_\beta\rangle$$
$$\xrightarrow{\text{a.l.}} \left\langle \Omega_\beta \right| \mathcal{S}(V)^{-1}\mathcal{S}(V + A_f) \left| \Omega_\beta \right\rangle = Z(f)$$

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The contour approach of Schwinger and Keldysh

Main idea

Accomplish the GML factorization by modifying the underlying spacetime:

$$Z(f) \xrightarrow{\text{a.l.}} \omega_\beta(\mathcal{S}_C(V) \cdot_{\mathcal{P}} A_f) = Z_C(f)$$

where the $\cdot_{\mathcal{P}}$ replaces time-ordering by path-ordering on some contour C in complexified Minkowski spacetime $\mathbb{C} \times \mathbb{R}^3$.

The contour C

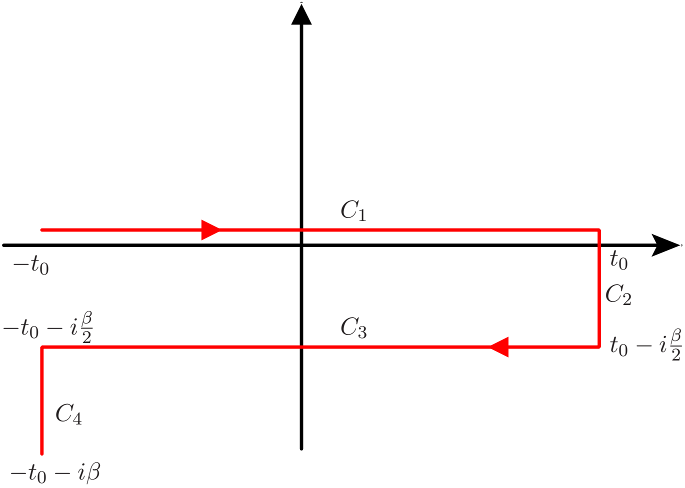

Figure: The Schwinger-Keldysh contour C, where the limit $t_0 \to \infty$ has to be taken.

Extension of the underlying spacetime

Spacetime defined by C

Denote by M_C the spacetime

$$M_C = \bigcup_{i=1}^{4} \mathcal{I}_i \times \mathbb{R}^3, \quad \mathcal{I}_i \subseteq \mathbb{R}$$

where the $\mathcal{I}_i$ are the parameter spaces of the individual contour pieces C_i.

Configuration space

Define the configuration space $\phi \in \mathcal{E}(M_C) \cong \bigoplus_{i=1}^{4} \mathcal{E}(M_{C_i})$, such that

$$\phi(\tau, \mathbf{x}) = (\phi_1(\tau_1, \mathbf{x}), \ldots, \phi_4(\tau_4, \mathbf{x})), \qquad \operatorname{supp} \phi_i \subset M_{C_i}, \ \tau_i \in \mathcal{I}_i$$

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Factorization

$\star$-product, $\cdot_{\mathcal{P}}$-product

Denote a parametrization of C by $\tau \mapsto C^i(\tau)$. Define $\star$ and $\cdot_{\mathcal{P}}$ for functionals on $\mathcal{E}(M_C)$ by the (4x4)-matrix-valued distributions

$$\mathbf{\Delta}^{ij}_{\bullet}(\tau,\tau') = \lim_{t_0\to\infty} \begin{cases} D^{\beta}_{+}(C^i(\tau) - C^j(\tau')) & i > j \\ D^{\beta}_{-}(C^i(\tau) - C^j(\tau')) & i < j \\ D^{\beta}_{\bullet}(C^i(\tau) - C^i(\tau')) & i = j \end{cases}, \quad \bullet \in \{F, +\}$$

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Theorem

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Equivalence of SK and TFD

Explicitly we obtain

$$\boldsymbol{\Delta}_+ = \begin{pmatrix} D_+^\beta & 0 & \tilde{D}_+^\beta & 0 \\ 0 & E_+^\beta & 0 & 0 \\ \tilde{D}_+^\beta & 0 & D_-^\beta & 0 \\ 0 & 0 & 0 & E_+^\beta \end{pmatrix}$$

$$\tilde{D}_+^\beta(t,0) = D_+^\beta(t + i\beta/2) \qquad t \in \mathbb{R}$$
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Isomorphic algebra

The subalgebra

$$\mathcal{A}_{13} = \left\{ A \in \mathcal{A}_C : \frac{\delta}{\delta\phi_2} A(\phi) = 0 = \frac{\delta}{\delta\phi_4} A(\phi) \right\}$$

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The S-matrix of theory is derived as the path-ordered exponential

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for an interaction $\bar{V} = \sum V_i$, supp $V_i \subset M_{C_i}$.

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The associated retarded operator R_V to $\mathcal{S}_C$ for an observable A is given by

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