Effective Quantum Plane from Quantum Physics

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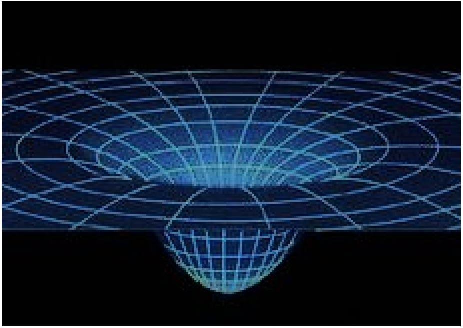
Outline

Motivation for physics on quantized spacetime

- Four known fundamental interactions. Standard model unifies three interactions. Attempts to unify gravity with the SM failed $\Rightarrow$ Quantum Gravity.
- Principles of QM+GRT: classical picture of spacetime breaks down near distances of the Planck length $\lambda_{Pl} = (\frac{G\hbar}{c^3})^{1/2}$.

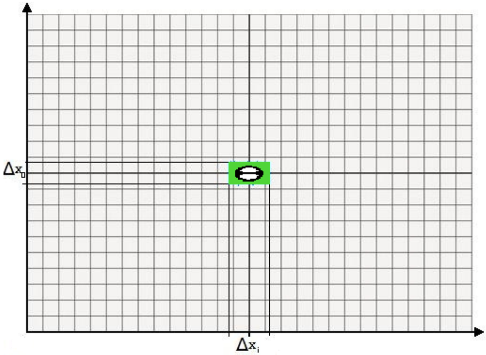


Replace coordinates x^μ by self-adjoint operators $\hat{x}^\mu$ obeying

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$$

$\Longrightarrow$ Uncertainty relations:

$$\Delta x^0 \Delta x^i \geq \theta/2$$



Aims for the talk

- Obtain quantum plane from QM and QFT

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- Find interpretation from theory for the deformation constant

Outline

X_μ selfadjoint and abelian $[X_\mu, X_\nu] = 0$,

$$\Longrightarrow \boxed{\exists \text{ strongly continuous unitary group } V(k) := e^{ik^\mu X_\mu}}$$

Buchholz, Lechner, Summers '10:

Definition of deformation

Let θ be a skew-symmetric matrix, then the warped convolution $A_{\theta,X}$ of $A \in C^\infty$ is

$$A_{\theta,X}\Phi := (2\pi)^{-d} \int\int dy\, dk\, e^{-iyk} V(\theta y)(A)V(-\theta y)V(k)\Phi, \qquad \Phi \in \mathcal{D} \subset \mathscr{H}$$

Deforming the Hamiltonian with X_j

Free Hamiltonian:

$$\boxed{H_0 = -P_j P^j/(2m)}$$

Deformed Hamiltonian with warped convolutions

$$H_{\theta,X}\Psi = -\frac{1}{2m}(P_j + \theta_{jk}X^k)(P^j + \theta^{jr}X_r)\Psi = -\frac{1}{2m}P_j^{\theta,X}P^j_{\theta,X}\Psi$$

The deformed Energy Eigenvalues

Quantized energy values for fixed p_1!

$$E_{\theta,n} = \frac{p_1^2}{2m} + \left(n + \frac{1}{2}\right)\frac{\theta}{m}, \qquad p_1 \in \mathbb{R}, n \in \mathbb{N}.$$

Physical Effects from Deformation

Landau levels

By setting $\theta = eB$ for deformed free Hamiltonian

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$\Longrightarrow$ Obtain any gauge field!

Obtaining Groups from Warped Convolution

Magnetic Translation Group

Generators of MTG

$$W(a) = e^{ia^j \pi_j}$$

with canonical momentum operator $\pi_i = P_i - eA_i$ satisfying the CR

$$[\pi_i, \pi_j] = -iF_{ij}.$$

Lemma

For $\theta_{ij} = -\frac{1}{2}F_{ij}$, the deformed generator of the Heisenberg-Weyl group $P^j_{\theta,X}$ is equal to the canonical momentum operator of the MTG π_i.

Quantum Plane from QM

In lowest LL motion described by $Q_i = X_i + (B^{-1})_{ik} P^k$,

$$\boxed{[Q_i, Q_j] = 2i(B^{-1})_{ij}.}$$

$\Longrightarrow$ Q_i spans quantum plane $\mathbb{R}^3_{2B^{-1}}$.

Lemma

$X_i^{\theta,P}$ satisfies the commutation relations of the Moyal-Weyl plane $\mathbb{R}^3_{-2\theta}$,

$$[X_i^{\theta,P}, X_j^{\theta,P}] = -2i\theta_{ij}.$$

If $-\theta_{ij}$ is $(B^{-1})_{ij}$, then $X_i^{\theta,P}$ are equal to the guiding center coordinates Q_i.

$\Longrightarrow$ Idea of lemma can be used in QFT!

Outline

ϕ obeys Klein Gordon Equation

$$\left(\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x_\mu} + m^2\right)\phi_0(x) = 0,$$

Solution:

$$\boxed{\phi_0(x) = \int\left(e^{-ipx}a(\mathbf{p}) + e^{ipx}a^*(\mathbf{p})\right)d^3\mathbf{p}/2\omega_{\mathbf{p}}}$$

where $\omega_{\mathbf{p}} = \sqrt{-p^i p_i + m^2}$.

Quantum plane from warped convolutions

Doplicher, Fredenhagen and Roberts '01

Realized scalar QF on NC Minkowski spacetime, represented on $\mathscr{V} \otimes \mathscr{H}$.

Grosse and Lechner '07

Represented NC counterpart of scalar QF, on $\mathscr{H}$ instead of $\mathscr{V} \otimes \mathscr{H}$.

Buchholz, Summers '08

Introduced warped convolutions and obtained GL QF by deformation with P_μ.

$\Longrightarrow$ **Idea** : Deform QF with FS operator to obtain quantum plane.

Special conformal operator K_μ

Commutation relations

$$[P_\rho, K_\mu] = 2i\,(\eta_{\rho\mu} D - M_{\rho\mu})\,, \qquad [K_\rho, M_{\mu\nu}] = i\,(\eta_{\rho\mu} K_\nu - \eta_{\rho\nu} K_\mu)\,,$$

$$[D, K_\mu] = iK_\mu, \qquad [K_\mu, K_\nu] = 0$$

Swieca, Völkel '76: Construct U_R in $\mathscr{H}_1$

$$\boxed{K_\mu := U_R P_\mu U_R}$$

$\Rightarrow \exists$. str. cont. unitary group $U(b) := e^{ib^\mu K_\mu}$

Deformation with special conformal operator K_μ *

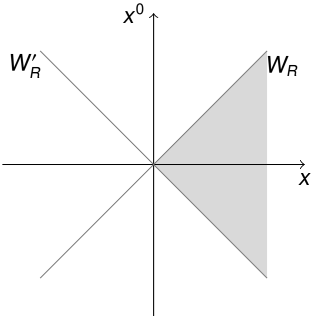
$$\boxed{\phi_{\theta,K}(f)\Psi := (2\pi)^{-d} \iint dy\, dk\, e^{-iyk} U(\theta y)\phi(f)U(-\theta y + k)\Psi, \qquad \Psi \in \mathcal{D}(K)}$$

- Deformed field fulfills the same bounds as undeformed field
- $\phi_{\theta,K}(f)$ satisfies Reeh-Schlieder theorem but not covariance and locality

Covariance and locality replaced by modified versions!

* AM, J. Math. Phys. 53, 082303 (2012)

Deformed fields defined as QF's on wedge by using map $Q : W \mapsto Q(W)$



Wedge-covariance and wedge-locality of $\phi_{\theta,K}$

Lemma

The special conformal transformations $U_{\theta v}$, with $v \in spU$ and θ being admissible, map the right wedge into the right wedge $U_{\theta v}(W_1) \subset W_1$.

Theorem

The family of fields $\phi = \{\phi_W : W \in \mathcal{W}_0\}$ defined by the deformation with K_μ is wedge-covariant, w.r.t. Lorentz group. Furthermore, for $n = 2l + 1$, where $l \in \mathbb{N}_0$, the field ϕ is a wedge-local field on $\mathscr{H}$.

Lemma

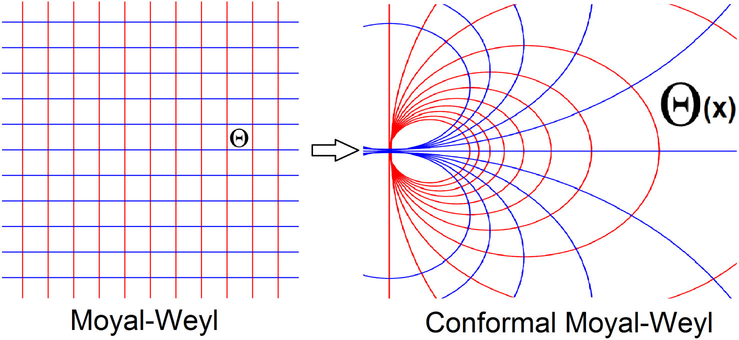
Let the deformed product be defined by K_μ. Then the deformed commutator up to third order in θ is

$$[x_\mu \overset{\times_\theta}{,} x_\nu] = -2i\theta_{\mu\nu}x^4 - 4i\left((\theta x)_\mu x_\nu - (\theta x)_\nu x_\mu\right)x^2 \equiv i\Theta_{\mu\nu}(x).$$

$\Rightarrow$ Nonconstant noncommutative spacetime.

Quantum plane from deformation with K_μ

$$[x_\mu \overset{\times_\theta}{,} x_\nu] = -2i\theta_{\mu\nu}x^4 - 4i\left((\theta x)_\mu x_\nu - (\theta x)_\nu x_\mu\right) x^2 \equiv i\Theta_{\mu\nu}(x).$$



Deformation of scalar field with operator R_μ

$$R_\mu := (D, M_{01})$$

Proposition

The family of fields $\phi = \{\phi_W : W \in \mathcal{W}_0\}$ defined by the deformation with R_μ is wedge-local, but **not** wedge-covariant.

Outline

Conclusion

- From QM obtained physical effects and physical quantum plane by deformation
- Deformation with SCOP Wightman field with wedge-covariant and wedge-local properties
- Quantum plane from SCOP deformation is nonconstant

Outlook

- Obtain Stark effect from deformation
- Scattering for deformed QFT's
- Deformation for Fermions and Gauge fields
- Generalize deformation to non Abelian groups
- ...


That's all Folks!