# The ongoing impact of modular localization on particle theory

To the memory of Hans-Jürgen Borchers (1926-2011)

Bert Schroer
present address: CBPF, Rua Dr. Xavier Sigaud 150,
22290-180 Rio de Janeiro, Brazil
email schroer@cbpf.br
permanent address: Institut für Theoretische Physik
FU-Berlin, Arnimallee 14, 14195 Berlin, Germany

April 2013

**Abstract**

Modular localization is the concise conceptual formulation of causal localization in the setting of local quantum physics. Unlike QM, it does not refer to individual operators but rather to ensembles of observables which share the same localization region; as a result it explains the probabilistic aspects of QFT in terms of the impure KMS nature arising from the local restriction of the pure vacuum. Whereas it plays no important role in perturbation theory, it becomes indispensable for understanding analytic and algebraic properties of on-shell objects as the S-matrix and formfactors.

This leads not only to a new critical evaluation of the dual model and string theory, but also identifies ideas of embedding and dimensional reduction as inconsistent with the holistic properties of localization. Instead it reveals the conceptual origin of true particle crossing and points the way to a new formulation of Mandelstam's on-shell project of the 60s.

Modular localization also shows that perturbative calculations in the Krein-space setting can be better done directly in Hilbert space with the help of short-distance lowering string-localized potentials. This points to a vast extension of renormalizability for any spin.

## Contents

# 1 Introduction

The course of quantum field theory (QFT) was to a large extend determined by three important conceptual conquests: its 1926 discovery by Pascual Jordan in the aftermath of what in recent times is often referred to as the *Einstein-Jordan conundrum* [1] [2] (a fascinating dispute between Einstein and Jordan), the discovery of renormalized perturbation in the context of quantum electrodynamics (QED) after world war II, and the non-perturbative insights into the particle-field relation initiated in the Lehmann-Symanzik-Zimmermann (LSZ) work on scattering theory which subsequently was derived from first principles [3] and applied to strong interactions in the context of the rigorous derivation of the particle analog of the Kramers-Kronig dispersion relations including their subsequent successful experimental test which extended the trust in QFT's foundational causality principle. These results encouraged a third project: particle-based on-shell formulations as the S-matrix bootstrap and Mandelstam's more analytic formulation in terms of auxiliary two-variable representations of elastic scattering amplitudes. The later gauge theory of the Standard Model resulted from an extension of the quantization ideas which already had led to QED. Besides many successes, it led to most of the still open problems of actual research.

Jordan's changed view with respect to Einstein's statistical mechanics argument in favor of the existence of photons [2] not only led him to accept Einstein's reasoning, but also encouraged him to extend his quantization also to matter waves. But its main point, the thermal character of subvolume fluctuation resulting from the restriction of the global vacuum state to the observables localized in that subvolume, did not receive the conceptual attention which being a characteristic property separating QFT from quantum mechanics (QM) it would have deserved. In fact the terminology "E-J conundrum" refers to this incomplete understanding [1]. The perception of the *stochastic thermal nature of the reduced vacuum state,* i.e. the fact that the restriction of the *pure* global vacuum state to observables localized in a subregion behaves as an impure KMS state was not understood; the E-J conundrum was forgotten in the maelstrom of time before its main lesson was learned.

The (by hindsight) obvious explanation is that for a very long time QFT was thought of as a relativistic form of QM with with infinite degrees of freedom. But in QM the vacuum does not become impure by spatial restriction, and this is independent of whether one uses the few degrees of freedom Schrödinger description or its infinite degree of freedom Fock space (second quantized) description. Many decades later when, in the special context of wedge-localization, this aspect of QFT was first noticed in form of an Gedankenexperiment[1] [4][5][6], the setting was too special and looked too contrived in order to suspect the existence of an insufficiently understood foundational structure. Even when historians directed their attention to the beginnings of QFT, they missed to see the analogy of the E-J conundrum with the "thermal" aspects of the Unruh effect; in [1] some of Jordan's calculations were presented in more details, but the purity of the ground state restricted to observables in a subvolume (its inside/outside tensor factorization property under spation bipartite separations) which holds in QM was also assumed to be true in QFT.

From a modern point of view the d=1+1 Jordan model of a chiral current (in his view a "2-dim. photon field") is the simplest illustration of "localization-caused thermal behavior" (LT) since it leads to a mathematical isomorphism [2] between LT and the global heat bath thermal behavior (HT) of statistical mechanics. In the Unruh Gedankenexperiment there is an *analogy* between LT and HT, but no isomorphism of the Unruh LT system to HT (i.e. no "inverse" Unruh effect in the sense of [7]), whereas Jordan's (in modern terminology) chiral current model leads to such an isomorphism.

Both Gedankenexperiments demonstrate a kind of "thermal" (see previous footnote) manifestation of causal localization whose early comprehension could have changed the path of QFT history. When Jordan's incomplete calculation was published as a separate section in the famous 1926 Dreimännerarbeit with Born and Heisenberg, his coauthors had some reservations, since it contained problematic aspects which had no place in the previously discovered QM; but they were not able to articulate their doubts.

Several years later Heisenberg challenged Jordan in a letter about a missing logarithmic term proportional to $ln\varepsilon$ in his calculation of the fluctuation spectrum where $\varepsilon$ is a length which characterizes the "fuzzyness" (i.e. the deviation from sharp localization) at the endpoints of Jordan's localization interval [1]. This led to Heisenberg's discovery of *vacuum fluctuation* near the localization boundaries with $\varepsilon$ the "attenuation length" $\varepsilon$ conceeded to the vacuum polarization cloud. As we know nowadays, the localization-caused vacuum polarization (VP) and LT are opposite sides of the same coin. In fact Jordan's missed logarithmically divergent localization entropy resulting in the sharp localization limit for $\varepsilon \to 0$ is the one-dimensional counterpart of the dimensionless area law $A/\varepsilon^2$ for *localization entropy* with $\varepsilon =$ "roughness" (attenuation length conceded to the VP cloud[2]) [9] [10].

[1] The impure KMS state in which the vacuum presents itself to an uniformly accelerated observer (respresenting a wedge-localized observable) does not imply that the "Carnot temperature" of a KMS state is the temperature measured with a thermometer. In the presence of external forces (acceleration) this relation breaks down, which has been overlooked in most of the literature [6]. LT irefers to a KMS impurity and not to a temperature in the sense of a thermometer which can be used for egg.boiling (Unruh).

[2] The infinities of entropy or energy in the sharp localization limits are consequences of the modular

These somewhat hidden properties of QFT place this theory into a sharp conceptual contrast to QM, it is neither QM with infinite degrees of freedom nor should it be referred to as "relativistic QM" [10]. In his famous paper on VP which he wrote after challenging Jordan about the missing $ln\varepsilon$ contribution, Heisenberg showed that the localization of dimensionless quantum charges ("partial charge") in QFT behaves quite different from their counterpart in QM. The inverse relation between sharpness of localization boundaries and increase of VP, measured in terms of the amount of entropy, is the QFT *substitute of the uncertainty relation* (which as the absence of the position operator among localized observables has no place in QFT). Relativistic QM built on the cluster factorization property but without the causal localization which deals directly with particles without the mediation of fields ("direct particle interactions") does exist, but besides a Poincaré-invariant S-matrix (no crossing property) it has none of the properties which characterize causal QFT [11].

The algebraic formulation of QFT, often referred to as *local quantum physics* (LQP) or algebraic QFT (AQFT), has brought the localization properties into the forefront by demonstrating that they have a natural *mathematical counterpart*: *the Tomita-Takesaki modular theory of operator algebras* [3]. The more recent terminology "modular localization" refers to its deep connection to the causal localization principle, which identifies QFT as *that* quantum theory (QT) which results from the mathematical implementation of this principle. It refers directly to localized subspaces and subalgebras instead of individual states and operators; the role of quantum fields is simply to "coordinatize" localized algebras by playing the role of pointlike *singular generators of all localized algebras*. Unlike QM, it does not refer to events related to *individual* operators, but rather deals with *ensembles* of operators (idealized as localized algebras) which share the same spacetime localization region. In consequence QFT leads via LT and the resulting KMS property to a statistical probability notion, the same "real" probability as used in statistical mechanics.

This is quite interesting from a conceptual point of view since Born's probability postulate in QM has been a point of philosophical controversies. The realization that probability is an unavoidable consequence of the quantum adaptation (of Einstein's Minkowski space formulation) of Faraday's and Maxwell's "action at the neighborhood principle" may have pleased Einstein, who had a lifelong problem with Born's assignment of probability to individual events (or to "imagined" ensembles) in order to interpret the QM. In section 3 and 4 some of its important definitions and consequences of modular localization will be presented.

The foundational role of modular localization begs the question: how was it possible to set up renormalized perturbation theory (the textbook QFT of Lagrangians quantization) without a thorough understanding of the foundational role of the causal localization principle and its consequences?

The answer is surprisingly simple: in overcoming the older quantum mechanical formulation (which missed the contributions from vacuum polarization), it was sufficient to implement the covariance requirement (Tomonaga, Feynman, Schwinger and Dyson), which is closely related to localization but not equivalent to it. The remaining problems took the form of consistent prescription of how to handle infinities in terms of cutoffs or

---

localization principle. In contrast to ultraviolet divergences in renormalization theory they are intrinsic and cannot be avoided by more appropriate formulation (the Epstein-Glaser distributional setting [12])..

regulators. Even later, after Epstein and Glaser [12] showed that an iductive use of causal locality which combined with a minimality requirement on the short-distance scaling limit leads to the renormalized result in a completely finite way, the above mentioned subtleties of causal localization still did not play a role. In retrospect one may say that modular localization only entered particle physics indirectly through Sewell's observation [5] that the identification of the modular group of wedge-localized algebras and the reparametrization in terms of the proper time of an accelerated observer accounts for the Unruh effect. But it took another three decades to unravel its constructive power.

Although modular localization had no direct impact on renormalized perturbation, it would be premature to conclude that its structural consequences are limited to E-J, the Unruh Gedankenexperiment and Hawking radiation. Recent conceptual progress in QFT revealed that LT explains the conceptual origin of the particle crossing property in on-shell quantities as the S-matrix and formfactors [10].

The particle crossing property was still unknown when Heisenberg attempted to formulate particle theory directly in terms of the $S_{scat}$ matrix [13] without referring to fields, whose insufficiently understood inherent singular character (Laurent Schwartz distributions) led almost to the rejection of QFT as a consistent description of relativistic interactions (the "ultraviolet catastrophe"). With the derivation of the LSZ scattering theory and certain analytic properties (needed in the derivation of the particle counterpart of the Kramers-Kronig optical dispersion relations) also the *crossing property* received attention. Through its perturbative identification in mass-shell restricted Feynman graphs, it became gradually clear that particle theory contained a somewhat mysterious analytic on-shell property, in which incoming particles became interchanged with outgoing antiparticles (after suitable analytic continuation). It was not possible to reduce this property to the known analytic properties of Wightman's [27] off-shell correlation functions (the Bargman-Wightman-Hall analytic domain). A rigorous derivation for special elastic scattering amplitudes from locality properties of off-shell 4-point functions was based on the use of the unwieldy mathematical theory of several complex variables [14]; as a result the conceptual origin of particle crossing remained a mysterious issue within the anyhow poorly understood field-particle relation beyond the LSZ scattering theory.

These incomplete attempts to unravel the nature of analytic particle crossing were mostly ignored in mainstream particle physics of the 60s; they did not fit the post QED Zeitgeist of S-matrix research in order to understand strong interactions, which mainly consisted in inventing computational rules and making analytic assumptions as the computations progressed. In retrospect it is clear that a foundational understanding of on-shell analytic properties from the causality principle of QFT was way beyond the conceptual knowledge at that time. As Heisenberg's first S-matrix attempt, also the bootstrap project came soon to a halt for the same reasons: the underlying principles were too general and the additional analytic working assumptions too vague and ad hoc in order to serve for the start of meaningful computations. Their nonlinear nature (unitarity "by hand" and not through the large time asymptotic scattering limit of linear field operators) created the wrong expectation that if the bootstrap admits a solution at all, it should be rather unique (a precursor of later "theories of everything").

Stanley Mandelstam [15], one of the most dedicated champions of a "top-to-bottom approach"[3] based on observable on-shell objects, tried to make the S-matrix project more amenable to calculations by adding reasonably-looking assumptions concerning two-variable spectral representation for the elastic scattering amplitude; in fact he introduced most of the kinematical on-shell terminology whose use became standard and will certainly outlast all other S-matrix ideas.

It is the main aim of the present work to show that this project was derailed from its original purpose of an S-matrix-based on-shell construction in particle theory when in the late 60s a crossing property based on mathematical properties of Euler's beta functions was proposed [16]. Its incorrect identification with particle crossing led to the dual model and finally to string-theory. The defining function of the dual model is a crossing-symmetric meromorphic function whereas the particle crossing in the S-matrix and formfactors is inconsistent with meromorphy in the Mandelstam variables; not even an approximand, which violates unitarity but maintains the other properties of particle scattering can be meromorphic (absence of cuts) in s,t,u, so that a "unitarization" of the dual model function does not help. This raises the question whether Veneziano's mathematical construction can be related with *any* property which one meets in QFT, or whether it remains the solution of an entirely mathematical game, as its origin suggests.

Following observations by Gerhard Mack [17][18], it will be shown in the next section that the meromorphic dual model crossing is a *rigorous property* of the Mellin transform of *conformal* 4-point-functions. The location of the poles of this function is given by the anomalous dimensional spectrum which in general has no bearing on particle physics. In fact not only Veneziano's dual model, but also all later versions are of this form. This somewhat unexpected property which looks like a particle pole contribution in an elastic scattering amplitude is what in the next section is called the "picture puzzle" appearance of the analogy between $(d, s)$ scale dimension spectra in conformal QFTs and the $(m^2, s)$ particle spectra in QFT with mass gaps. Since there are no known QFTs with infinite particle spectra which preserve the cardinality of degrees of freedom associated to the physical causal completeness property, this analogy places question marks on the consistency of such infinite mass/spin towers with the principle of causal localization.

Therefore it is important to be more explicit about the mathematical properties of this "picture puzzle relation" between a local conformal world in which the spectrum of scale dimensions is necessarily infinite, and a local particle world in which an infinite particle spectrum is ruled out because too many degrees of freedom lead to a breakdown of the causal completeness property. Its resolution will be that the *existence of a positive energy Poincaré representation on the irreducible oscillator algebra of a particular 10 dimensional chiral conformal current model* indeed exists. This group theoretic fact may be surprising, but it bears no relation to interacting particles and their S-matrix; the picture of an associated interacting "target" model is simply incorrect. It is a result of a "picture puzzle" situation whose resolution will be presented in the next section.

In section 4 we return to the problem of the true origin of particle crossing in the S-matrix and formfactors and its use in on-shell constructions; it will be shown that the particle crossing identity is nothing else as *the KMS property* associated with wedge localization and rewritten in terms of *emulated free-field associated particle states (* a new

[3] An alternative of the standard "bottom-to-top quantization" with its intermediate ultraviolet problems in which the physical interpretation (top) starts after the end of the computations.

concept from modular localization). It is very pleasing that the recognition of the failure of the old S-matrix approach is also the start of a new S-matrix-based on-shell construction (section 4), this time based on modular localization.

As a preparatory step for section 4, one needs to know some basics about modular operator theory. This is the purpose of *section 3,* which starts by explaining the limitations of the standard way of covariantizing Wigner's positive energy representation and how modular localization of wave functions helps to overcome them. The modular localization of subspaces prepares the ground for the modular localization of operator algebras which in turn leads to the Tomita-Takesaki modular theory in the LQP setting of operator algebras [3].

Historically the idea of modular localization of wave functions entered Wigner's representation theory as the result of trying to understand the resistance of the zero mass infinite spin class (faithful representation of Wigner's "little group") against any attempt to extract a covariant field from those representations. This problem was only solved more than 6 decades after Wigner's pathbreaking work with the help of modular localization [19][20] by realizing that this class of representations only admits semiinfinite string-localized, instead of pointlike generating wave function. This explained immediately why there was no classical analogue i.e. no Lagrangian from to which this wave function was related through a Euler-Lagrange variation[4]. Allowing string-like solutions also turned out to resolve the well-known clash of massless pointlike vectorpotentials with the Hilbert space positivity. The better alternative between the two possibilities (either pointlike in Krein space or stringlike in Hilbert space) is the latter. The same remedy applies to higher spin massless representation.

In section 3 it is also shown how this observation leads to a radical changes of the concept of renormalizability with a new view about remaining foundational problems of the Standard Model. In contrast to the second section which takes a critical look at more than 5 decades of dual model/string theory, the third section focusses on the ongoing radical changes about renormalization theory involving higher spins ($s \geq 1$) with consequences for the Standard Model.

The concluding remarks address problems which may explain the deep schism between particle physics in the critical tradition as pursued by a minority (as represented in section 3 and 4 in the present paper) and the more metaphoric ST-influenced majority view of what constitutes particle theory whose historical origin had been critically reviewd in section 2.

Our findings support the title and the content of a contribution by the late Hans-Jürgen Borchers in the millennium edition of Journal of Mathematical Physics [21] : "Revolutionizing Quantum Field Theory with Tomita-Takesaki's modular theory". With all reservations about misuses of the word "revolution" in particle physics, this paper is a comprehensive account of the role of modular operator theory in LQP and its historical origin. Its title is a premonition of the present progress which is driven by concepts coming from modular localization. LQP ows Borchers many of the ideas coming from modular operator theory; for this reason it is very appropriate to dedicate the present article to his memory.

[4] As a rule of thumb (consistent with all that is known): string-localized fields are not Euler-Lagrange and Euler-Lagrange objects (ST a la Polyakov) are not string-localized.

The crucial new insight which permits to view this new setting also as a legitimate heir of Mandelstam's S-matrix ideas before ST, is the observation that the S-matrix, in addition to its well known role in scattering theories, is a *relative modular invariant between the wedge-localized interacting algebra and its free counterpart* (generated by the incoming free fields smeared with wedge-supported test functions). This new role of the S-matrix was already implicitly contained in Res Jost's work on the TCP theorem in the setting of a complete particle interpretation; but it only found its first constructive application after it was realized that the Zamolodchikov-Faddeev algebra operators of integrable d=1+1 models *are the generators of spacetime-localized wedge algebras* in the setting of integrable QFTs [22][23].

Some historical remarks may facilitate to understand the aims of this paper which deals with foundational problems of QFT which developed over many decades. The natural conceptual framework in which the modular localization attained its important role is the algebraic LQP setting of QFT. It started with Haag's 1957 attempt[5] [25] to base QFT on *intrinsic principles* instead of subordinating a more fundamental theory via a *quantization parallelism* to a less fundamental classical field theory. The idea that a foundational theory as QFT should not be forced to "dance to the tune" (Jordan used the expression "classical crutches") coming from a less fundamental classical theory can already be found in some of Jordan's early work [26], but the necessary algebraic concepts were not yet available at his time. Hence the terminology LQP in the present work stands for a *different formulation* of QFT *which maintains its physical content.* Another setting, which also does not refer to quantization, was Wightman's [27] formulation of quantum fields in terms of operator-valued Schwartz distributions and their correlation functions. The two approaches are conceptually closely related by viewing the Wightman fields as generators of local algebras.

The quantum aspects of causal localization and the associated maximal propagation speed have been the cause of innumerous misunderstanding. Even one of the most reputable research journals published an article in which Fermi's famous Gedankenexperiment to demonstrate that the classical limitation through the velocity if light passes to QED was thrown into doubt [31]. Following critical remarks, PRL also published the correct arguments [32]. The "effective" localization of propagating (spreading) wave packets in QM (e.g. the velocity of sound) is not changed in QFT apart from the fact that there is a maximal effective velocity. But different from QM[6], the localized observables of QFT retain the *exact* (in contrast to effective) classical relativistic propagation properties in the form of *modular localization* of LQP. But the latter now lead to completely different nonclassical consequences: fields as singular objects (operator-valued distributions), VP near localization boundaries ($\rightarrow$ localization entropy) and KMS properties of spacially restricted vacuum states, including the natural appearance of probability without Born's interpretive addition to QM. The rather comprehensive correct account in PRL [32] did however not stop the later appearance of "superluminal" papers in other journals, whose error can always be traced back to the same misunderstanding of causal localization.

---

[5] The original version is in French even though most of the talks were in English. Later it was translated back into English in [24]..

[6] As classical mechanics QM lacks the "interaction at the neighborhood", but its wave functions admit an effective finite propagation speed.

The presentation of results is strictly limited to their mathematical-conceptual content; only in the concluding section I permit myself some remarks about their position in the sociological-ideological struggle of the search of the "heart and soul" of particle theory; the origin of the present schism is explained as the result of a lack of balance between innovative proposals and their critical evaluation within the new on-line globalized community of particle theory and its charismatic leaders as compared to those few groups and individuals who uphold the traditional conduct.

# 2 Anomalous conformal dimensions, particle spectra and crossing properties

A large part of the conceptual derailment of string theory can be understood without invoking the subtleties of modular localization. This will be the subject of the following four subsections.

The principle of *modular localization* becomes however essential for a foundational understanding of the particle crossing property which is important for a new formulation of a constructive on-shell project based on the correct crossing property which replaces Mandelstam's S-matrix setting with an on-shell construction which is compatible with the principles of Haag's local quantum physics. This will be the subject of section 4.

## 2.1 Quantum mechanical- versus causal- localization

Since parts of the misunderstandings in connection with ST have to some extend their origin in confusing "Born localization" in QM with the causal localization in QFT, it may be helpful to review their significant differences [11].

It is well-known since Wigner's 1939 description of relativistic particles [3] in terms of irreducible positive energy representations of the Poincaré group, that *there are no 4-component covariant operators* $x_{op}^{\mu}$; in fact the impossibility to describe relativistic particles in terms of quantizing a classical relativistic particle action was one of the reasons for the representation theoretical construction of their wave function spacesn by Wigner. The rather simple argument against covariant selfadjoint $x_{op}^{\mu}$ follows from the non-existence of covariant spectral projectors $E$

$$\begin{aligned} \vec{x}_{op} &= \int \vec{x} dE_{\vec{x}},\ R \subset \mathbb{R}^3 \to E(R) \qquad (1) \\ U(a)E(R)U(a)^{-1} &= E(R+a),\ E(R)E(R') = 0\ for\ R \times R' \\ (E(R)\psi, U(a)E(R)\psi) &= (\psi, E(R)E(R+a)U(a)\psi) = 0 \end{aligned}$$

where the second line expresses translational covariance and orthogonality of projections for spacelike separated regions. In the third line we used that suitable translation shifts $E(R)$ spacelike to itself. But since $U(a)\psi$ is analytic in $\mathbb{R}^4 + iV^+$ ($V^+$ forward light cone) as a result of the spectrum condition, $\|E(R)\psi\|^2 = 0$ for all $R$ and $\psi$ which implies $E(R) \equiv 0$ i.e. *covariant position operators do not exist.*

The "Born probability" of QM results from Born's proposal to interpret the absolute square $|\psi(\vec{x}, t)|^2$ of the spectral decomposition $\psi(\vec{x}, t)$ of state vectors with respect to the spectral resolution of the position operator $\vec{x}_{op}(t)$ at time t as a probability density. Its use as a probability density to find an individual particle in a pure state at a prescribed position became the beginning of one of a still not closed philosophical dispute in QM which Einstein entered through his famous saying: "God does not throw dice".

In Haag's LQP setting this problem does not exist since, as previously mentioned, its objects of interests are not global position operators in individual quantum mechanical systems, but rather ensembles of causally localized operators which share the same spacetime localization i.e. belong to the spacetime-indexed algebras $\mathcal{A}(\mathcal{O})$ of Haag's LQP (next section). The modular localization attributes statistical mechanics-like KMS properties resulting from a highly impure *reduced* vacuum state to such an ensemble. As in statistical mechanics, the KMS property *is inherited by all the individual operators* of $\mathcal{A}(\mathcal{O})$ without invoking Born's interpretative postulate about squares of wave functions[7].

Traditionally the causal localization of QFT enters the theory with the (graded) spacelike commutation (Einstein causality) in Minkowski spacetime of pointlike localized covariant fields. There are very good reasons to pass to another slightly more general, but in a subtle sense also more specific formulation of QFT, namely to Haag's local quantum physics (LQP) in which the fields play the more auxiliary role of (necessary singular) generators of local algebras[8]. In analogy to coordinates in geometry there are infinitely many such generators which generate the same algebra as there are different coordinates which describe the same geometry. As in Minkowski spacetime geometry these "field coordinates" can be chosen globally i.e. the same generating field for the generation of all local algebras associated to one LQP.

In this more conceptual LQP setting it is easier to express the *full* content of causal localization in a precise operational form. It includes not only the Einstein causality for spacelike separated local observables, but also a timelike aspect of causal localization, namely the equality of an $\mathcal{O}$-localized operator algebra $\mathcal{A}(\mathcal{O})$ with that of its *causal completion* $\mathcal{O}''$

$$\begin{aligned} &\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}''),\ \mathcal{O}' = causal\ disjoint\ of\ \mathcal{O},\ causal\ completeness \qquad (2) \\ &\mathcal{A}(\mathcal{O}') \subseteq \mathcal{A}(\mathcal{O})',\ \ = is\ Haag\ duality, \subset\ \ Einstein\ causality \end{aligned}$$

(with $\mathcal{A}(\mathcal{O})'$ commutant of $\mathcal{A}(\mathcal{O})$).The causal completeness requirement does not follow from Einstein causality and corresponds to the classical causal propagation. A closely related property is Haag duality. The advantage of the LQP formulation over the use of fields is clearly seen in case of these two properties.

It is not evident at all that this timelike causal completion aspect of causality is intimately related to the cardinality of phase space degrees of freedom. Whereas both properties are *formal* attributes of Lagrangian quantization, they have to be added in "axiomatic" settings based on mathematically controlled (and hence neither Lagrangian nor functional) formulations [28]. Their violations for subalgebras $\mathcal{A}(\mathcal{O})$ as a result of too

[7] As mentioned there exists still the hope to derive Born:'s probability in QM as a relic of the intrinsic LQP probability in a conceptually better understood future limit of QFT in which the modular localization itself is lost.

[8] To be more precise they are operator-valued Schwartz distributions whose smearing with $\mathcal{O}$-supported test functions are (generally unbounded) operators affiliated with a weakly closed operator algebra $\mathcal{A}(\mathcal{O})$.

many phase space degrees[9] of freedom leads to physically undesirable effects, which among other things limit the physical application of the mathematical AdS-CFT correspondence (last subsection).

On the other hand the violation of Haag duality for disconnected or multiply connected regions have interesting physical consequences in connection with superselection sectors associated with observable algebras and also with the QFT Aharonov-Bohm effect for doubly connected spacetime algebras which has its simplest formulation in (m=0,s=1) Wigner representations with possible generalizations to multiply connected spacetime regions in higher spin (m=0,s>1) representations [33][34].

The LQP formulation of QFT is naturally related to the Tomita-Takesaki modular theory of operator algebras; its general validity for spacetime localized algebras of the latter is a direct result of the Reeh-Schlieder property [3] for localized algebras $\mathcal{A}(\mathcal{O}), \mathcal{O}'' \subset \mathbb{R}^4$ (next section).

It is important to understand that quantum mechanical localization is not cogently related with spacetime. A linear chain of oscillators simply does not care about the dimension of space in which it is pictured; in fact it does not even care if it is related to spacetime at all or whether it refers to some internal space to which spacetime causality concepts are not applicable. The modular localization on the other hand is *imprinted* on causally local quantum matter, it is a totally *holistic* property of such matter. As life cannot be explained in terms of the chemical composition of a living body, localization does not follow from the mathematical description of the global oscillators (annihilation/creation operators) in a global algebra. These oscillators are the same in QM and QFT; free field oscillator variables $a(p), a^*(p)$ which obey the oscillator commutation relations do not know whether they will be used in order to define Schrödinger fields or free covariant local quantum fields. It is the holistic *modular localization principle* which imprints the causal properties of Minkowski spacetime (including the spacetime dimension) on operator algebras and thus determines in which way the irreducible system of oscillators will be used in the process of localization [35]; in QFT there is, strictly speaking, no abstract quantum matter as there is in QM; rather localization becomes an inseparable part of matter. Contrary to a popular belief (the credo about dimensional reduction and extra dimensions), this holistic aspect of QFT (in contrast to classical theory and Born's localization in QM) *does not permit an embedding of a lower dimensional theory into a higher dimensional* one, neither is its inversion (Kaluza-Klein reduction) possible

One problem in reading articles or books on ST is that it is sometimes difficult to decide which localization they have in mind. When e.g. Polchinski [36] uses the relativistic particle action $\sqrt{ds^2}$as a trailer for the introduction of the Nambu-Goto minimal surface action $\sqrt{A}$ (with $A$ being the quadratic surface analog of the line element $ds^2$) in a description of ST, it is not clear why he uses this as an analogy to a quantum string; as a trailer to a relativistic quantum string this is based on a genuine misunderstanding a kind of conceptual "squib load"[10].

The Polyakov action $A$ can be formally written in terms of the potential of an n-component chiral current

[9] For the notion of phase space degree of freedoms see [98][29][30]

[10] Relativisic covariant particles cannot be obtained by quantizing particle Lagrangian; one either must use Wigner's representation theoretical approach or the indirect route through QFT of free fields.

$$\int d\sigma d\tau \sum_{\xi=\sigma,\tau} \partial_\xi X_\mu(\sigma,\tau) g^{\mu\nu} \partial^\xi X_\mu(\sigma,\tau) \tag{3}$$
$$X = potential\ of\ conformal\ current\ j$$

However the quantum theory related to the Nambu-Goto action has nothing to do with its square (see later). On the other hand the use of the letter $X$ for the potential of the multicomponent chiral current suggests that Polchinski's quantum mechanical trailer has taken roots in the incorrect idea that the action of a multi-component massless field describes in some way a covariant string embedded into a higher dimensional Minkowski spacetime, similar to the (correct idea) of an embedding of a linear chain of oscillators into a higher dimensional QM.

If the quantized $X$ of the Polyakov action would really describe a covariant spacetime string, one could forget about the N-G square root action and take the Polyakov action for the construction of an embedded string. But this cannot work since the principle of modular localization simply contradicts the idea that a lower dimensional QFT can be embedded into a higher dimensional one. In particular an n-component chiral conformal QFT cannot be *embedded* as a "source" theory into a QFT which is associated with a representation of the Poincaré group acting on the n-component inner symmetry space (the "target" space) of a conformal field theory. What is however possible for the supersymmetric oscillators contained in a d=10 supersymmetric chiral current model is to construct a highly reducible positive energy Wigner representation of the Poincaré group.

String theorists gave a correct proof of this group theoretic fact, but it cannot be used to justify an embedding a string into a 10 parametric picture. In fact the abstract irreducible global oscillator algebra admits at least two inequivalent representations: one on which the Möbius group acts and in which it is possible to construct pointlike Möbius covariant covariant fields, and the other on which the mentioned unique 10-dimensional representation of the Poincaré group acts and which leads to infinitely many irreducible pointlike generated wave function spaces (or by second quantization to an infinite component free field). The easiest way to see that the representations are different is to notice that the multi-component charge spectrum is continuous and the corresponding Poincare momentum spectrum has mass gaps. In addition the embedding picture would suggest that the object is a spacetime string and not an infinite component pointlike wave function (field). The group theoretic theorem cannot be used in an on-shell S-matrix approach; To construct an S-matrix one needs more than just group theory.

Of course the mentioned group theoretic is somewhat surprising since it is the only known irreducible algebra which leads to a discrete mass/spin tower (without admixture of a continuous energy-momentum spectrum). Often one obtains a better understanding by generalizing a special situation. Instead of an irreducible algebra associated with a chiral current theory one may ask whether an internal symmetry space of a finite component quantum field can (i.e. not indices referring to spinor/tensor components of the field) carry the representation of a noncompact group. In classical theories this is always possible, whereas in QFT one would certainly not expect this in d>1+1 models. For theories with mass gaps this is the result of a very deep theorem about the possible superselection structure of observable LQP algebras [3] and there are good reasons to believe that this continues to hold in theories containing massless fields [37]. A necessary prerequisite is the existence of continuously many superselected charges as in the case of abelian current models. By definition this is the class of non-rational chiral models. Apart from the multicomponent abelian current model, almost nothing is known about this class, so the problem whether the "target spaces" of such models can accommodate unitary representations of noncompact groups i.e. the question whether the above theorem about unitary representations on multicomponent current algebras is a special case of a more general phenomenon remains unknown.

A rather trivial illustration of a classical theory on whose index space a Poincaré acts without the existence of a quantum counterpart is the afore-mentioned relativistic classical mechanics. As covariant classical theories may not have a quantum counterpart, there are also strong indications about the existence of QFTs which cannot be pictured as the quantized version of classical fields[11]. Up to now the setting of QFT within the quantization setting was large enough to account for all observational relevant models, but one should be prepared for future changes.

The best way of presenting the group theoretical theorem discovered by string theorists is to view it in a historical context as the (presently only known) solution of the 1932 Majorana project [38]. Majorana was led to this project by the $O(4,2)$ group theoretical description of the nonrelativistic hydrogen spectrum. We take the liberty to formulate it here in a more modern terminology.

**Problem 1** *(Majorana) Find an irreducible algebraic structure which carries a infinite-component positive energy one-particle representation of the Poincaré group (an "infinite component wave equation").*

Majorana's own search as well as that for the so-called "dynamic infinite component field equation" of the 60s (Fronsdal, Barut,...;see appendix of [39]) consisted in looking for irreducible group algebras of noncompact extensions of the Lorentz group ("dynamical groups"), but no acceptable solution was ever found within such a setting. The only known solution is the above superstring representation which results from an irreducible oscillator algebra of the n=10 supersymmetric Polyakov model. The positive energy property of its one-particle content and the absence of components of Wigner's "infinite spin" components (which cannot be pointlike generated) secures the pointlike localizability of this "superstring representation".

The misunderstanding about localization in this terminology is a reminder that the subtleties of the quantum causal localization principle, nowadays incorporated into the modular localization setting of LQP, took a long way from the Einstein-Jordan conundrum and many other confusions caused by retaining relics of the non-intrinsic quantum mechanical localization in problems of QFT. In view of the fact that relativistic QM exists but bears no relation to causal propagation (although it carries a representation of the Poincaré group its only covariant operator is the S-matrix [11]), such a terminology is very misleading. The use of quantum mechanical notation $X_\mu(\sigma,\tau)$ in ST is bound to

[11] There are many known d=1+1 integrable models which have no known Lagrangian description.

create confusions about localization because the conceptual content of symbols is often identified with their past use.

The confusions about localization often did not enter the calculations of string theorists but remained in the interpretation. A poignant illustration is the calculation of the (graded) commutator of string fields in [40][41]. Apart from the technical problem that infinite component fields can not be tempered distribution (since the piling up of free fields over one point with ever increasing masses and spins leads to a diverging short distance scaling behavior which requires to project onto finite mass subspaces), the commutator is pointlike. Certainly this uncommon distributional behavior has no relation with the idea of spacetime strings; at most one may speak about a quantum mechanical chain of oscillators in "inner space" (over a localization point). The memory of the origin of ST from an *irreducible* oscillator algebra is imprinted in the fact that the degree of freedoms used for the representation of the Poincaré group do not exhaust the oscillator degrees of freedom, there remain degrees of freedom which interconnect the representations in the (m,s) tower i.e. which prevent that the oscillator algebra is only a direct sum of wave function spaces. But the localization properties reside fully in these wave function spaces and, as a result of the absence of Wigner's infinite spin representations, the localization is pointlike. This is precisely what the above-mentioned authors found, but why did they not state this clearly, why did they instead talk about a point on a (imagined) string? Has Heisenberg's admonition to limit quantum physics to observables been dismissed in order to serve an ideology?

Does the bizarre suggestion that we are living in an dimensionally reduced target space of an almost unique[12] 10-dimensional chiral conformal theory become more acceptable if it continues in the less bizarre but nevertheless incorrect form of embeddings of causal localizable QT i.e. in the believe that there exists a well defined geometric relation between theories of different spacetime dimensionality (embeddings and dimensional reductions)? The answer is a clear no; the ideas of Kaluza and Klein originated at a time when the foundational difference between QM and QFT were not yet noted. Such ideas may be consistent with theories which do not possess an *intrinsic* notion of localization (and its subtle connection with phase space degrees of freedom) as quasiclassical approximation or QM, but they clash with the holistic aspect of modular localization which imprints the spacetime dimensionality onto causal quantum matter.

The main point of this article is to convince the unbiased reader that indeed a sizable part of the particle theory community has moved into an increasingly metaphoric direction instead of solving the hard problems of localization which existed since the time of the Einstein-Jordan conundrum and only surfaced gradually in the LQP setting of QFT. Although the errors of ST are known to most physicists familiar with the LQP who tried to understand ST from a conceptual point of view, it is not possible to overcome the present schism on this point by a rapid transfusion of LQP acquired knowledge about modular localization; the split happened already many decades ago and became solidified within globalized communities. Actually one can assign an exact date to the beginning of metaphoric particle physics, it was the day of the proposal of the dual model and its subsequent widespread acceptance as an S-matrix property.

[12] Up to a finite number of M-theoretic modifications.

It is understandable that this mathematically sophisticated model had a hypnotic effect on high energy phenomenologists which at the time were looking for descriptions of infinite particle trajectories. As a result of its rich mathematical content it also attracted more conceptually oriented physicists who thought that such deep mathematical observations deserves a connection with a more foundational kind of physics than the phenomenological "reggeology". The phenomenological excitement was cooled down by new unsupportive observational results, but a critical assessment of ST on the theoretical conceptual side did not happen. To the contrary, there were comments as "ST is a gift of the $21^{th}$ century which by luck fell into the $20^{th}$ century and similar statements by reputable physicists, and even many decades later no serious attempt to critically compare ST with the sucessful on-shell construction of integrable d=1+1 QFT; the few attempts to understand the origin and nature of particle crossing of S-matrices and formfactors from the causal localization principle were initially partially successful, but then got stuck in the messy details of the theory of several complex variables [14].

Res Jost was the last physicist who used his deep conceptual understanding of QFT and its relation to S-matrix properties in order to criticise the bootstrap S-matrix approach[42]. A critique of ST is more subtle and has, according to my best knowledge, never before been undertaken with the necessary conceptual mathematical precision. Part of the reason may be that the endurance of ST over so many decades is related to a somewhat confusing picture puzzle between (m,s) spectra of QFTs with mass gaps and (d,s) spectra in conformal QFT. In fact the solution of the Majorana project in terms of a an infinite component discrete (m,s) spectrum of a 10-dimensional chiral current theory is an illustration of this picture puzzle aspect (see below). Another reason is that ST had so many physical and philosophical weaknesses that the time it takes to find its conceptual errors was not considered worthwhile to invest, especially since the string theorists enjoyed considerable mathematical support. In fact up to this date only mathematicians obtained valuable progress from ideas from ST which they succeeded to make precise in a way which suits them. Part of the reason why ST was not analyzed from a foundational viewpoint (as in this article) is certainly related to the support from the mathematical side.

Our criticism of the dual model and ST is two-fold, on the one hand we will remind the reader that that the meromorphic crossing of the dual model, although not related to particle theory, is a rigorous property (no "unitarization" possible) of conformal correIations after generating s,t,u variables which resemble Mandelstam's scattering variables via Mellin transformations. The poles in these variables occur at the scale dimensions of composites which appear in global operator expansions of two conformal covariant fields. The spacetime dimensionality does not play any role, any conformal QFT leads to a dual model and that found by Veneziano belongs to a chiral current model. A special such spectrum appears if one asks the question whether the abstract oscillator algebra underlying a chiral conformal current theory can support a unitary representation of the Poincaré group. The answer to this question is much more restrictive and corresponds to a very special dual model case. In this case the $(m^2, s)$ Poincaré spectrum is proportional to the dimensional spectrum $(d, s)$ of composites in the global operator expansion which led to the pole spectrum of the dual model.

One can phrase this important observation in a historical setting: the superstring representation of the Poincaré group is the only solution of Majorana's 1932 project to find an infinite component relativistic field equation from an irreducible representation of an (any) algebra. One may consider this as a surprising result, but it is purely group theoretic and has no bearing on S-matrix theory. It also has no bearing on embedding a "string" (a chiral conformal theory on the lightray) into a higher dimensional theory which, as was already mentioned, is not allowed anyhow by the holistic properties of modular localization.

The second criticism of ST amounts to showing how particle crossing arises from the principle of modular localization. This does not only reveal the difference to dual model crossing, but also suggests new on-shell construction methodes based on the S-matrix which is capable to replace Mandelstam's approach.

## 2.2 The picture puzzle of chiral models and particle spectra

There are two ways to see the correct mathematical-conceptual meaning of the dual model and (what for historical correctness is called) ST without being side-tracked by treacherous analogies.

One uses the "Mack machine" [17][18] for the construction of dual models (including the dual model which Veneziano constructed "by hand"). One starts from an conformal 4-point function of *any conformal QFT in any spacetime dimension.* To maintain simplicity we take the vacuum expectation of four not necessarily equal scalar fields

$$\langle A_1(x_1)A_2(x_2)A_3(x_3)A_4(x_4)\rangle \tag{4}$$

It is one of the specialities of interacting conformal theories that fields have no associated particles with a discrete mass, instead they carry a (generally a non-canonical, anomalous) scale dimensions which are connected with the nontrivial *center of the conformal covering group* [10]. It is well known from the pre BPZ [43] conformal research in the 70s [44] [45] that conformal theories have converging operator expansions of the type

$$A_3(x_3)A_4(x_4)\Omega = \sum_k \int d^4z \Delta_{A_3,A_{4.},C_k}(x_1,x_2,y)C_k(z)\Omega \tag{5}$$

$$\langle A_1(x_1)A_2(x_2)A_3(x_3)A_4(x_4)\rangle \rightarrow 3\ different\ expansions \tag{6}$$

In distinction to the Wilson-Zimmermann short distance expansions which only converge in an asymptotic sense, these expansions converge in the sense of state-vector-valued Schwartz distributions. The form of the global 3-point-like expansion coefficients is completely fixed in terms of the anomalous scale dimension spectrum of the participating conformal fields; i.e. unlike in models with a particle interpretation, one does not have to dive deeply into the dynamics in order to get a rather explicit understanding of the operator expansions and their coefficient functions.

It is clear that there are exactly three ways of applying global operator expansions to pairs of operators inside a 4-point-function 6, analogous to the three possible particle pairings in the elastic S-matrix which correspond to the s,t and u in Mandelstam's formulation of crossing. But beware, this dual model crossing arising from the Mellin transform of conformal correlation has nothing to do with S-matrix particle crossing of Mandelstam's on-shell project ! If duality would have arisen in this context probably nobody would have connected it with the particle crossing in S-matrices and on-shell formfactors. But Veneziano found it [16][46] from properties of the Euler beta function which did not reveal its conformal origin. Since particle crossing and its conceptual origin in the principles of QFT remained somewhat hidden (for a recent account of its origin from modular localization see [47][10]), the identification of crossing with Veneziano's duality met little resistance. As mentioned, it could have been clear with a bit more hidsight that it has no relation to particle crossing since the S-matrix cannot be meromorphic in Mandelstam's variables (and cannot even be approximated in this way); many useful messages could alreay have been learned from the rigorous construction of integrable models which have no inelastic processes.

The Mellin transform of the 4-point-function on the other hand is a meromorphic function in s,t,u which has first order poles at the numerical values of the (generally) anomalous dimensions of those conformal composites which appear in the three different decompositions of products of conformal fields; they are related by analytic continuation [17][18]. To enforce an interpretation of particle masses, one may rescale these dimensionless numbers by the same dimensionfull number. However this formal step of calling the scale dimensions of composites particle masses does not change the physical reality. Structural analogies in particle physics are worthless without an independent support concerning their physical origin.

The Mack machine to produce dual models (crossing symmetric analytic functions of 3 variables) has no definite relation to spacetime dimensions; one may start from a *conformal theory in any spacetime dimension* and end with a meromorphic crossing function in Mellin variables. Calling them Mandelstam variables does not change the conceptual-mathematical reality which for scattering amplitudes (unitarity, inconsistency of particle crossing which are meromorphic in Mandelstam variables) is totally different from that (öf Mellin transforms) of conformal correlation (exact meromorphy, "unitarization" is meaningless); one is dealing with two quantum objects whose position in Hilbert space can hardly be more different than scattering amplitudes and conformal correlations; no unitarization scheme can mathematically change one into the other.

However, and here we come to the picture-puzzle aspect of ST, one can ask the more modest question whether one can view the *dimensional spectrum of composites* in global operator expansions (after multiplication with a common dimensionfull $[m^2]$ parameter) *as arising from a positive energy representation of the Poincaré group.* The only such possibility which was found is the previously mentioned 10 component superymmetric chiral current theory which leads to the well-known superstring representation of the Poincaré group and constitutes the only known solution of the Majorana project[13]. In this way the analogy of the anomalous composite dimensions of the poles in the dual model from the Mack machine to a $m^2$ mass spectrum is extended to a genuine particle representation of the Poincaré group. But even this lucky circumstance, which leads to the superstring representation, remains on the level of group theory and cannot be viewed as containing dynamic informations about a scattering amplitude; not even in an

[13]To see this, the representation theory of the irreducible oscillator algebra of the chiral current model is more suitable than the Mack machine.

approximate sense.

There exists a presentation which exposes this "picture-puzzle" aspect between conformal chiral current models and particle properties in an even stronger way: the so-called sigma-model representation. Schematically it can be described in terms of the following manipulation on abelian chiral currents ($x =$ lightray coordinate)

$$\partial\Phi_k(x) = j_k(x),\ \Phi_k(x) = \int_{-\infty}^{x} j_k(x),\ \langle j_k(x) j_l(x')\rangle \sim \delta_{k,l}\left(x - x' - i\varepsilon\right)^{-2} \tag{7}$$
$$Q_k = \Phi_k(\infty)\,,\ \Psi(x,\vec{q}) = \text{"} : e^{i\vec{q}\vec{\Phi}(x)}\text{"} :\ ,\ carries\ \vec{q} - charge$$
$$Q_k \simeq P_k,\ dim(e^{i\vec{q}\vec{\Phi}(x)}) \sim \vec{q}\cdot\vec{q} \simeq p_\mu p^\mu,\ (d_{sd}, s) \sim (m, s)$$

The first line defines the *potentials of the current*; it is formally infrared-divergent and should not be used to generate the vacuum sector which is created from the vacuum by applying the polynomial algebra generated by the current alone. In contrast *the exponential sigma field* $\Psi$ *is the formal expression for a covariant superselected charge-carrying field* whose symbolic exponential way of writing leads to the *correct correlation functions only in total charge zero correlations* where the correlation functions agree with those computed from Wick-reordering[14] of products of : $expiq\Phi(x)$ :,all other correlations of this "sigma" field vanish (the quotation mark in (7) indicates this limitation of the formal notation).

The interesting line is the third in (7), since it expresses a "mock relation" with particle physics in which the multi-component continuous charge spectrum of the conformal currents resembles the continuous momentum spectrum of a representation of the Poincaré group whereas the spectrum of anomalous scale dimensions (being quadratic in the charges) is reminiscent the quadratic relation between momenta and particle masses. The above analogy amounts to a genuine positive energy representation of the Poincaré group for the special case of a supersymmetric 10-component chiral current model; it is the before-mentioned solution of the Majorana project; but its appearance in the Mellin transform of a conformal correlation has nothing to do with an S-matrix. As also mentioned, the shared irreducible abstract oscillator algebra leads to different representations in the use for the conformal theory and the localization which is related to the positive energy representation of the Poincaré group[15]. The difference between the representation leading to the conformal chiral theory and that of the Poincare group on the target space (the superstring representation) prevents the (structurally anyhow impossible) interpretation in terms of an embedding of QFTs; although there remains a certain closeness as a result of the shared oscillator algebra.

The multicomponent $Q_\mu$ charge spectrum covers the full $\mathbb{R}^{10}$ whereas the $P_\mu$ spectrum of the superstring representation is concentrated on positive mass hyperboloids; the Hilbert space representation of the algebraic oscillator substrate in order to obtain localization and Möbius invariance on the light ray is not the same as that which leads to that

[14]The two-point function of $\Phi$ being the indefinite metric logarithm of x-x'.is indefinite but the exponential correlations together with the charge-conservation coply with the Hilbert space structure.

[15]The 26 component model does not appear here because we are interested in localizable representation; only positive energy representations are localizable.

of the superstring representation. Hence presenting the result as an embedding of the chiral "source theory" into the 10 component "target theory" is a metaphoric exaggeration having its psychological origin probably in the picture-puzzle aspect; the representation theoretical differences express the different holistic character of the two different localizations (the target localization being a direct consequence of the intrinsic localization of positive energy representations of the Poincaré group). What remains is a mathematical question: why does the positive energy representation of the Poincaré group only occur when the chiral realization has a vanishing Virasoro algebra parameter? And are there other non-rational (continuous set of superselection sectors) chiral models which solve the Majorana project? Both questions can be genalized to: are their other nonrational chiral theories with (discrete sums of irreducible) representations of inner noncompact symmetries (target representations) ?

It should be added that it would be totally misleading to reduce the mathematical/conceptual role of chiral abelian current models to their picture puzzle use in ST, or their role in the solution of the Majorana project. The chiral n-component current models played an important conceptual role in mathematical physics; the so-called *maximal extensions* of these observable algebras can be classified by even integer lattices and the possible superselection sectors of these so extended algebras are classified in terms of their duals [48][49][50]. Interestingly the selfdual lattices and their relation with exceptional final groups correspond precisely to the absence of non-vacuum superselection sectors which in turn is equivalent to the validity of *full* Haag duality (Haag duality also for all multiply-connected algebras [33][34]). They constitute the most explicitly constructed nontrivial chiral models which shed light on the interplay of discrete group theory and Haag duality as well as on violation of Haag duality for disconnected localization regions and anyon statistics and many other surprising consequences of modular localization. This is more than a consolation for their inability to reveal properties about higher dimensional scattering amplitudes.

## 2.3 General structural arguments against embeddings and dimensional reductions

The important property which permitted to associate the representation of a noncompact group (in the above case the Poincaré group) with the "target" of a chiral QFT, is the existence of superselected charges with a continuous spectrum. This is only possible in chiral theories, more specifically in nonrational (by definition) models; the only known such model is the one presented in the previous subsection.

This cannot occur in a higher dimensional theory with a complete particle interpretation since the DHR superselection theory (and its Buchholz-Fredenhagen [51] extension) leads to a compact group of inner symmetries [3]. The idea of inner symmetries, which dates back to Heisenberg's nuclear isospin, is a quantum notion which in the quantization approach to QFT is "red back" into the classical Lagrangian field formalism. However classical Lagrangians are also consistent with the action of noncompact symmetry groups. A trivial example is the mentioned classical covariant path Lagrangian $\sqrt{ds^2}$ whose Euler-Lagrange equation is the covariant description of a classical particle which has no quantum counterpart; whether the classical covariant surface solutions of the N-G Lagrangian admit a covariant quantum counterpart is very questionable. Quantization is not a principle, rather it is a conceptional limited, but observational successful artistic device; not every covariant classical theory has a covariant quantum counterpart, neither can one expect that a QFT, which has been constructed in an intrinsic way (see the algebraic construction in section 4), can be described in terms of Lagrangian quantization.

The concept of causal localization is too holistic in order to permit an embedding or a dimensional reduction outside of quasiclassical approximations of QFT, it would contradict the principle of modular localization. Nobody has ever been able to show that the correlation functions of a model in lesser spacetime dimension can be obtained by restricting a higher dimensional QFT, nor that the inverse association of two QFTs by embedding is possible. The lack of any intrinsic structural argument (which according to the modular localization aspect of LQP cannot exist) did not prevent the appearance of thousands of papers and the creation of special sections in journals and at conferences. This has grown into a sociological/psychological bulwark which seems to be impenetrable to scientific arguments. Existing "proofs" of the Kaluza-Klein mechanism in QFT are always based on "massaging" Lagrangians or manipulations in terms of quasiclassical approximations, but such arguments ignore what happens with genuine quantum degrees of freedom in such manipulations. A closely related issue is that of *branes*; in that case Mack [17][18] has shown that in passing from the full theory to a brane, there is *no thinning out of degrees of freedom.* This preservation of cardinality of phase space degrees of freedom leads to an non acceptable causality violation (violation of the causal completeness property, the "poltergeist phenomenon").

This causality violation is the same as that occurring in the mathematical AdS-CFT correspondence. If one starts from a locally causal AdS model, the associated CFT will be unphysical as a result of that poltergeist-causing violation of causal completeness; and in case one starts from a physical CFT, the resulting AdS model whose existence is guaranteed by the correspondence will be too "anemic". In fact its compact double cone algebras has no degrees of freedom at all ( i.e. those algebras are trivial, being generated by the unit operator); only in noncompact spacetime regions extending to infinity (as wedges) degrees of freedom will be present [52]. The Maldacena conjecture, which presumes that both sides of the correspondence are "physical", plainly contradicts these known facts.

Observations about the relation between the independence of the causal completeness property from the Einstein causality started in the early 60s [28]; the use of generalized free fields also indicated the relation with too many degrees of freedom. Later this connection between the cardinality of degrees of freedom and causal localization was sharpened, first to compactness and afterwords to "nuclearity" [3].

## 2.4 The correct implementation of quantization for the N-G action

The classical geometric surface embedding as defined by the N-G action and treated according to the rules of reparametrization invariant quantum systems poses a similar problem of diffeomorphism invariance as the quantization of the Einstein-Hilbert action [53]. This is intimately connected with the physical problem of implementing *background independence* in both of these cases. Even though the E-H and the N-G actions are nonrenormalizable (i.e. its perturbative calculations leads to an increasing with perturbative order set of undetermined parameters), there are arguments that the issue of background independence can be discussed independent of the renormalizability issue [54] (in which case the principle of background independence cannot be used to restrict the increasing number of parameters .

The problems of the N-G quantization and its diffeomorphism covariance has been recently treated in [53]. The application of this computational setting to the $\sqrt{ds^2}$action results apparently in the quantum theory of a nonrelativistic particle[16] and there is no reason that its N-G counterpart has anything to do with a *covariant* QT in the sense of a representation of the Poincaré group. So it seems that only the canonical quantization of the Polyakov action can be associated with a representation of the Poincaré group which solves, as explained before, the Majorana project, but has no relation to an on-shell construction of the kind Mandelstam was looking for.

It is hard to imagine that string theorists would be satisfied with such a group theoretic result, but it is the only mathematical fact which can be salvaged from the Mandelstam on-shell construction project after the incorporation of Veneziano's dual model and ST. In the section 4 we will present the derivation of particle crossing from the modular properties of wedge-localization. This does not only show that there is no relation to the crossing used in ST, but it also leads to a new formulation of an on-shell construction project which may be considered as a extension of what Mandelstam had in mind before the appearance of the dual model.

Before closing this subsection it may be interesting to mention another more concrete attempt to explore the physical content of the quantum N-G model. This is due to Pohlmeyer [55] who established the *existence of infinitely many classical conservation laws,* which suggest that the model is integrable. For integrable models there exists a more intrinsic way of quantization which is based on the Poisson bracket structure between the globally conserved quantities. Such a quantization has a higher degree of plausibility than canonical quantizations (which anyhow do not refer to the N-G action but rather to its Polyakov square). In a series of interesting publications Pohlmeyer and his collaborators studied the quantization of the Poisson-bracket relations between the conserved quantities. The drawback from the point of view of the intentions of the ST community is that this does not reveal anything about (point or stringlike) localization and its possible covariant behavior under Poincaré transformation.

Since these were rigorous mathematical results about the quantum theory of the N-G action, Pohlmeyer also called his approach to this model "string theory" even though possible relations to spacetime localization remained unresolved since exact global conservation laws do not contain any information about localization in contrast to ST which decomposes into infinitely many fields sitting "on top" of a localization point.

After having reminded the reader that the group theoretic content at the core of ST is the point-like localizable 10-dimensional "superstring" representation of the Poincaré group, he may be curious to learn how genuine string-localized objects really look like and what is their expected physical role in particle theory; this will be the content of the next section.

[16] I am indebted to Jochen Zahn for informing me on this point.

# 3 Higher spin interactions and modular localization

Wheras the previous section consisted in a foundational critique of 4 decades of ST and its derivatives, the main topic of this third section will be the presentation of modular localization and its use in showing the short-distance improvements which converts hitherto nonrenormalizable pointlike higher spin interactions into renormalizable interactions of their stringlike counterparts. In the first subsection we start with a short review about the connection between positive energy representations of the Poincaré group and the construction of point-localized covariant fields similar to Weinberg's method of covariantizing Wigner's representations [56]. This subsection also introduces a pedestrian description of string-localized free fields as well as a schematic description of their ongoing use in the enlargement of renormalizability to interactions involving arbitrary high spins.

A conceptual/mathematical backup in terms of modular localization is the task of the second subsection, whereas the third subsection is meant to indicate the enormous potential of these ideas in the ongoing and future Standard Model research.

## 3.1 Wigner representations and their covariantization

Historically the use the new setting of modular localization started with a challenge which remained since the days of Wigner's particle classification: *the causal localization of the third Wigner class* (the massless infinite spin class) of positive energy representations of the Poincaré group. Whereas the massive as well as the zero-mass finite helicity class are pointlike generated, it is not possible to find covariant pointlike generating wave functions for this third Wigner class. The first representation theoretical argument suggesting the impossibility of a pointlike generation dates back to [57]. It was followed decades later by the concept of modular localization of wave functions [19][22] which led to the introduction of spacelike string-generated fields in [20]. These are covariant fields $\Psi(x,e)$, $e$ spacelike unit vector, which are localized $x+\mathbb{R}_+e$ in the sense that the (graded) commutator vanishes if the full semiinfinite strings (and not only their starting points $x$) are spacelike separated [20]

$$[\Psi(x,e),\Phi(x',e')]_{grad}=0,\ \ x+\mathbb{R}_+e\ \rangle\langle\ x'+\mathbb{R}_+e' \tag{8}$$

Unlike decomposable stringlike fields (pointlike fields integrated along spacelike halflines) such *elementary stringlike fields* lead to serious problems with respect to the activation of (compactly localized) particle counters. Whereas known stringlike localized *fields*[17] cannot only create pointlike generated massive or zero mass finite helicity representations, the zero mass infinite spin representations consist of stringlike localized *states*. This represents a much more radical situation than the well-known clash between pointlike localization of vectorpotentials and the Hilbert space positivity (below).

In the old days [56] infinite spin representations were rejected on the ground that nature does not make use of them. But whether in times of dark matter one would uphold such dismissals is questionable. As the result of their irreducible semiinfinite localization as states they cannot be measured in compactly extended counters and it is

[17]The best known stringlike fields are the charge-carrying fields in QED (later subsection). These "infraparticle" fields create states from the vacuum which, upon continuous resolution into irreducible Wigner representations, only contain ordinary massive and zero mas representation.

doubtful that such matter can interact with "normal" matter; if it would not be for the property of being massless, they would be the ideal candidates for dark matter [33][34]. String-localized quantum fields fluctuate both in $x$ as well as in $e$[18]. They can always be constructed in such a way that their effective short distance dimension is the lowest possible one allowed by positivity, namely $d_{sd} = 1$ *for all spins*.It is very difficult to construct the covariant "infinite spin" fields by the group theoretic intertwiner method used by Weinberg [56]; in [20][58] the more powerful setting of modular localization was used. In this way also the higher spin string-localized fields were constructed.

For finite spins the unique Wigner representation always has many covariant pointlike realizations. In the spinorial description $\Psi^{(A,\dot{B})}(x)$ one finds the following relation between the spinorial $(A,\dot{B})$ characterization and the physical spin $s$ or helicity $h$

$$\left|A-\dot{B}\right| \leqslant s \leqslant A+\dot{B},\ m>0 \tag{9}$$

$$h = A-\dot{B},\ m=0 \tag{10}$$

In the massive case all possibilities for the angular decomposition of two spinorial indices are allowed, whereas in the massless case the values of the helicities $h$ are severely restricted (10). For $(m=0, h=1)$ the formula conveys the impossibility of reconciling pointlike vector potentials $\Psi^{(\frac{1}{2},\frac{1}{2})}\tilde{}A_\mu$ with the Hilbert space positivity. This clash occurs for all $(m=0, s\geq 1)$ pointlike localized "field strengths" (in h=2, the linearized Riemann tensor,..) have no pointlike quantum "potentials" (in h=2, the $g_{\mu\nu}$,...), and similar statement holds for half-integer spins in case of $s>1/2$. Allowing *stringlike generators*, the possibilities of massless spinoral $A,\dot{B}$ realizations are identical to those in the first line (9).

Since the classical theory does not care about positivity, (Lagrangian) quantization inevitably forces for $(m=0, s\geq 1)$ the *abandonment of the Hilbert space in favor of Krein spaces* (implemented by the Gupta-Bleuler or BRST formalism). The more intrinsic Wigner representation theoretical approach keeps the Hilbert space and lifts the restriction to pointlike generators in favor of covariant semiinfinite stringlike generating fields.

It is worthwhile to point out that perturbation theory does not require the validity of Lagrangian/functional quantization. Actions which lead to Euler-Lagrange quantization limit the covariant realizations of (m,s) Wigner representations to a few spinorial/tensorial fields with low $(A,\dot{B})$, but as Weinberg already emphasized for setting up perturbation theory one does not need Euler-Lagrange equations; they are only necessary if one uses formulation in which the interaction-free part of the Lagrangian enters as in the Lagrangian/functional quantization. The only "classical" input into causal perturbation as the E-G approach is a (Wick-ordered) polynomial which implements the classical notion of pointlike locality, all subsequent inductive steps use quantum causality and renormalizability. In the modular localization based setting of section 4 even this last weak link with classical thinking is lost and one enters the area of LQP without "classical crutches".

For $(m=0, s=1)$ the stringlike covariant potentials $A_\mu(x,e)$ are uniquely determined in terms of the field strength $F_{\mu\nu}(x)$ and a spacelike direction $e$. The idea is somewhat

[18] These long distance (infrared) fluctuations are short distance fluctuation in the sense of the asymptotically associated d=1+2 de Sitter spacetime.

related to Mandelstam's early attempt to formulate QED without the vectorpotentials [15]. But even though the string-local potential is uniquely determined in terms of $F_{\mu\nu}$ and $e$ , it is not possible to implements the reduction of the short distance singularity without the introduction of the covariant $A_\mu(x,e)$ (represented by a semiinfinite line integral over the field strength along a semiinfinite line in the direction $e$) as an object in its own right because in this way one cannot overlook that one is dealing with *objects which fluctuate in both* $x$ *and* $e$; in fact the improvement of the short distance property in $x$ is paid for by a worsening but still well-defined infrared behavior i.e. the $A_\mu(x,e)$ is an *operator-valued distribution in both* $x, e$. In contrast to the above infinite spin representation which cannot be obtained from integrating a pointlike object (and which for this reason behaves like (see previous remark) "zero mass dark matter"[19]), all other zero mass representations admit pointlike generators and only exclude pointlike potentials.

As an illustrative example for the use of those objects, let us look at the Aharonov-Bohm effect in QFT[20]. In terms of Haag's intrinsic LQP setting of QFT this amounts to a breakdown of *Haag duality* (2) *for a toroidal spacetime localization* [33][34]

$$\mathcal{A}(\mathcal{T}') \subsetneq \mathcal{A}(\mathcal{T})' \tag{11}$$
$$\mathcal{T} \;\; \textit{spatial torus at } t=0 \; , \; \mathcal{T}\,'' \textit{ its causal completion}$$

For lower spin zero mass fields or for a torus-localized algebra from a massive field of any spin, one finds the equality sign (Haag duality). This can be shown in terms of field strengths, but if one (for the convenience of applying Stokes theorem) uses the indefinite metric potentials one gets the wrong result, namely equality (zero effect). On the other hand the use of the string-localized potential in the Hilbert space accounts correctly for the A-B effect as the breakdown of Haag duality for multiply connected spacetime regions. It is expected that the breakdown of Haag duality for multiply connected regions is a general feature of higher spin zero mass representations.

In massive theories there is no such clash between localization and Hilbert space and there is also no violation of Haag duality in multiply connected regions. Pointlike potentials exist in Hilbert space (e.g. the Proca vectorpotential), but their short distance dimensions increase with spin just like those of field strengths (example: $d_{sd}(A_\mu^P) = 2$). Nevertheless one can introduce *stringlike potentials as a means to lower the short distance dimension* in order make the couplings fit for renormalization. The connection between the covariant[21] stringlike vectorpotential and its pointlike counterpart (the $d_{sd} = 2$ *Proca* potential $A_\mu^P(x)$) leads to a scalar string-localized field, the counterpart of the Stückelberg field

$$A_\mu(x,e) = A_\mu^P(x) + \partial_\mu \phi(x,e) \tag{12}$$
$$d_e A_\mu(x,e) = \partial_\mu \phi(x,e), \;\; d_e \phi(x,e) = \textit{pointlike scalar}$$

Here $d_e$ is the $e$-projected derivative with respect to $e$. Note that here the scalar Stückelberg field is not an independent field (as it would be in the BRST setting) i.e. the string

[19] Whereas the commutator of stringlike vectorpotentials behaves rather regular as long as the x-points of the strings $x+\mathbb{R}_0^+ e$ rdo not coalesce, for the infinite spin strings the crossing of their lines with specelike separated m $x's$ leads already to singularities.

[20] The standard A-B effect is about quantum mechanical charged particle in an *external* magnetic field.

[21] The spacelike string-direction e participates as a vector in the covariance law.

description of free fields has also *mixed* two-point functions between $A_\mu(x,e)$ and $\phi(x,e)$; the physical Hilbert space, in which also the string-localized fields are living, is fully determined in terms of $A_\mu^P(x)$ and the $e$-dependent covariant fields are relatively string-local (same Borchers class) with respect to the Proca field[22]. **The pointlike scalar resulting from the $d_e$ operation resembles a Hermitian scalar as that of the Higgs mechanism, but here it is not an additional degree of freedom but contained in $A_\mu(x,e)$ (see last subsection).**

The role of the new adiabatic equivalence requirement consists in showing that the formally nonrenormalizable interaction with the physical Proca is in perturbative precise sense equivalent to that of the better behaved string-localized potential. The guiding principle is that the undesired short distance singular terms which distinguish the the two intersctions can be collected into surface terms which in massive theories vanish in the adiabatic limit; the germ of this idea is of course the relation of the two vectorpotentials through the derivative of the string-dependent Stückelberg field (12). The nonperturbative LQP interpretation of the adiabatic equivalence is of course that the pointlike field is relative local (member of the same Borchers class) to the string-localized field.

The verification uses the Bogoliubov formula for the perturbative physical S-Matrix and the physical fields. For massive QED the interaction density $\mathcal{L}$

$$\mathcal{L}(x,f)=\int def(e)\mathcal{L}(x,e),\ \mathcal{L}(x,e)=A_\mu(x,e)j^\mu(x),\ \mathcal{L}=\int g(x)\mathcal{L}(x,f)dx \tag{13}$$

$$\psi_{int}(x,f):=\frac{\delta}{i\delta h(x)}S(\mathcal{L})^{-1}S(\mathcal{L}+h\psi)|_{h=0},\ S(\mathcal{L})=Te^{i\int g(x)\mathcal{L}(x,f)dx}$$

leads, according to the formal Bogoliubov prescription, to the perturbative S-matrix as well as to fields (indicated for the simplest case in the second line) for the interacting Dirac spinor; time-ordered products of interacting fields originate from higher functional derivatives[23]. The physical S-matrix results from the Bogoliubov $S$-functional in the adiabatic limit $g(x)\equiv 1$. The existence of this limit is only guarantied in the presence of mass gaps. The physical interacting fields $\psi_{int}^{phys}(x,f)$ also require this adiabatic limit; but as a result of the appearance of the inverse $S$-functional, the requirement for their existence are less stringent. They are localized in a spacelike cone with apex $x$ and require the same renormalization treatment as a pointlike d=1 field.

The reason why the smearing function in the string direction can be fixed, is that it plays a different role from $g$ since no limit has to be performed on the $f$. The resulting physical $\psi(x,f)$-field depends nonlinearly on $f$ and is localized in a spacelike cone with apex at $x$[24], but whose distributional extension problem still follows the iterative E-G scheme in which only the remaining counterterm liberty is still determined by the total diagonal in the apices [63]. The physical content of the theory can be extracted from the spacelike cone localized fields for fixed $f$-smearing since the LQP description of the particle-field relation also works for spacelike cone localization [3].

The important aspect to notice in connection with string-localized fields with finite spin is that their Wigner particle representations *always* admit covariant potentials which

[22] If one reads the equation as a definition of $A^P$, one easily shows $d_e(A_\mu-\partial_\mu\phi)=0$

[23] In order to include field strengths one needs another source term i.e. $S(L+h\psi+kF)$.

[24] The apex is also the point which is relevant for the Epstein-Glaser distributional continuation.

have the lowest possible $x$-singularity which accounts for their short distance dimension d=1 [20] and thus permits interactions below the power-counting bound *for all spins.* In contrast to pointlike realizations they achieve this improved short distance behavior by "spreading" the difference between the $d_{point}$ which increases with $s$ to $d_{string} = 1$ "over the string" which accounts for the fact that, although the string localization is seen in the commutation relations (8) of the potentials, the counterterm freedom of E-G renormalization is still described by pointlike terms.

In fact the main new idea used in the ongoing research on this problem is that certain formally pointlike nonrenormalizable couplings (e.g. interactions involving massive vectorpotentials) are "adiabatically equivalent" to renormalizable stringlike formulations. This important new insight also amounts to the possibility to compute a point-like perturbation series via the round-about way of doing renormalization theory in the string-like setting and afterwords passing via adiabatic equivalence to the pointlike correlation functions. In this way an old idea [60] to go beyond Schwartz distributions to what is now known as "hyperfunctions" in order to enter the area of nonrenormalizability takes on new actuality. Mathematicians found an interesting subset of hyperfunctions which still admit dense sets of compactly localizable test functions [61] which Arthur Jaffe [62] identified as "strictly localizable fields" (SLF) into QFT, showing among other things that the exponential of a free field and some of those fields used in couplings between massive vectorpotentials and scalar fields in [60] belong to this class. There are clear indications that the kind of nonrenormalizability of pointlike physical (BRST-invariant) matter fields in massive vectormeson couplings are of this SLF kind, which explains why they were not accessible in a pointlike renormalizable Hilbert space setting without the detour via string localization.

Taking into account the short-distance scaling degree of free massive string-localized potentials $d_{string} = 1$ instead of $d_{point} = 2$ for pointlike potentials (Proca), the formulation of the adiabatic equivalence principle starts with establishing the following first and second order relation ($\mathcal{L}^P$ Proca Lagrangian $d_{s.d.} = 5$)

$$\mathcal{L} = \mathcal{L}^P + \partial_\mu V^\mu,\ V^\mu(x,e) \equiv j^\mu(x)\phi(x,e),\ \mathcal{L}' \equiv \mathcal{L}(x_2,e_2) \tag{14}$$
$$T\mathcal{L}\,\mathcal{L}' - \partial_\mu T\ V^\mu \mathcal{L}' - \partial_\nu' T\mathcal{L}\ V^{\nu\prime} + \partial_\mu\partial_\nu' T V^\mu V^{\nu\prime} = T\mathcal{L}^P\mathcal{L}^{P\prime}$$

The last relation is a formal second order relation between the string- and the point-like description. It is trivially satisfied for the first term in the Wick expansion of time-ordered products. It is reasonably easy to check in the tree approximation. To fulfill it on the level of one-loop loop term is more demanding. The pointlike nature of the tree and loop terms is established by showing that the directional derivative $d_e$ and $d_{e'}$ vanish[25]. For details we refer to a forthcoming paper by Jens Mund [63]. The fact that scalar "massive QED" has quadratic terms in the vectorpotentials does not lead to new problems [64]. Note that the crucial point of the adiabatic equivalence is that the[26] difference between the nonrenormalizable pointlike and the renormalizable stringlike formulation consists of derivative terms which vanish in the adiabatic limit; the high dimensional terms which rendered the pointlike formulation nonrenormalizable are flushed away to infinity.

[25] The E-G extension of the loop term is quite tricky, but the resulting counterterm freedom is still pointlike [63].

[26] Part of joint project together with Jens Mund and Jakob Yngvason.

New problems however arise in Yang-Mills couplings as a consequence that the equation which prepares the implementation of the adiabatic equivalence become nonlinear in higher orders (color indices omitted)

$$A_\mu(x,e) = U(\phi(x,e))A_\mu^P(x)U(\phi(x,e))^* + \partial_\mu\phi(x,e) \tag{15}$$

Here the color components of $\phi(x,e)$ multiplied with the coupling function $g$ play the role of the numerical parameters in the $U$-color rotation. In this approach there is no gauge principle in addition the the fundamental causal localization principle. Behind this formula (assuming that the associated adiabatic equivalence can be verified, which for the nonabelian case still has to be done) is again the implementation of the adiabatic equivalence principle i.e. in the present context the verification that the that the formally nonrenormalizable pointlike interaction involving a Proca field defines in the adiabatic limit the same QFT as that based on the interaction with the corresponding stringlike vectorpotential [27]. For the first time it appears that one has an intrinsic access to the gauge concept in which the umbilical cord to classical physics has been completely cut and everything is viewed as a realization of the modular localization principle.

The nonlinear part of this formula already contributes to the second order in addition to those contributions which come from the trilinear and quadrilinear selfinteractions of the vectorpotentials. It would be desirable to show that the $U$ are exponential SLF fields in the sense of Jaffe, but this is not necessary for the perturbative use of (15) for the implementation of the adiabatic äquivalence. This calculation, which may decide over whether there is a theoretical necessity for the coupling to neutral scalar multiplets (Higgs) fields, has not been done at the time of writing of the present paper.

There exists another approach to massive vectormeson coupling, the BRST formalism. The formula which relates the Proca potential to a dimension d=1 BRST potential is similar to the above

$$A_\mu^{BRST}(x) = A_\mu^P(x) + \partial_\mu\phi^{BRST}(x) \tag{16}$$

where $\phi^{BRST}$ is the indefinite metric Stückelberg field. In this case the lowering of $d_{s.d.}$ from 2 to 1 is the result of the Krein space setting. But whereas in the string-localized description the matter-field exists as a spacelike cone-localized object (which is still a distribution in $x$), the existing literature does not contain a prescription for obtaining *physical matter fields* within the BRST formalism but is presently restricted to the S-matrix [65] of massive vectormesons. Therefore it is interesting to note that the application of the adiabatic equivalence suggests a way to solve this open problem.

After having addressed some technical question concerning the renormalization process for abelian massive vectormesons, there remains the important question of the existence of pointlike physical fields. To be more explicit, this amounts in both cases (string-localization and the BRST formulation in Krein space) to the question of the status of that pointlike matter-field whose short distance dimension increases with increasing perturbative order. The only nonperturbative requirement on such a field is that it is relative local to the string-localized respectively pointlike BRST fields. The existence of

[27] In the global adiabatic limit the interacting Proca field and the matter field to which it couples are relatively local (member of the same Borchers class); hence are different coordinatizations of the same QFT.

such singular but yet localizable objects is strongly suggested[28] by the following *adiabatic equivalence* relation for the generating Bogoliubov S-functional

$$S(\mathcal{L}+h\psi+kF)\simeq S(\hat{\mathcal{L}}+h\hat{\psi}+kF) \tag{17}$$
$$\hat{\mathcal{L}}=\mathcal{L}(A_{\mu}(x,e)\rightarrow A_{\mu}^{P}(x)),\ \hat{\psi}(x)=e^{-i\phi(x,f)}\psi(x,f)$$
$$or\ for\ BRST:\ \hat{\psi}(x)=e^{-i\phi(x)}\psi^{BRST}(x),\ if\ \mathcal{L}\rightarrow\mathcal{L}^{BRST}$$

where the $\simeq$ stands for equality in the adiabatic limit $g(x)\rightarrow 1.$ The transformation to the new S-functional is a formal operator gauge transformation inside the same theory which leaves $F$ unchanged and transforms $\psi$ into the (nonrenormalizable) pointlike candidate $\hat{\psi}$ for the the matter field in the adiabatic limit. The adiabatic equivalence of the BRST functional with that formally obtained from the nonrenormalizable pointlike formulation in a Hilbert space is the corresponding statement in case of the Krein formulation; in this case all fields remain formally point-localized. But whereas the string-like formulation allows a massless limit, *the pointlike BRST formulation has no massless physical* $\hat{\psi}$ *limit* (there are simply no infrared finite pointlike fields to which stringlike fields could be adiabatically equivalent) and the nonexistence of a massless analog[29] Proca field as a result of the clash between pointlike localization and Hilbert space.

Since the string-localized massless vectorpotentials of the Hilbert space formulation are uniquely fixed in terms of the field strength $F_{\mu\nu}(x)$ and the spacelike string direction $e$, the input is the same as in Mandelstam's attempts to formulate QED solely in terms of field strengths. It turns out that precisely the directional fluctuation of the $x+\mathbb{R}_{+}e$ localized $A_{\mu}(x,e)$ in $e$ (a point in d=1+2 de Sitter spacetime) attenuate the strength of the x-fluctuations and renders the interaction renormalizable in the sense of power-counting. The picture is that the nonvanishing commutators for string crossing are necessary for lowering the singularity for coalescent $x's$. Mandelstam's approach probably failed because in his setting it seems to be difficult to take care of this advantage [59]. In both, the massless as well as the massive case, there always exists a string-localized description in which the $e$-fluctations lower the strength of the $x$-fluctuation in the pointlike description in such a way that the resulting short distance scale dimension is d=1 independent of spin.

This deeper understanding, which is unfortunately blurred behind the widespread accepted vernacular "long distances are outside of perturbation theory"[30], leads to the recognition that the correctly formulated massless perturbation approach (using the stringlike nature of fields) avoids these off-shell infrared divergence problems in the standard formulation of Yang-Mills couplings. The only remaining genuine infrared problem is the question of how to relate perturbatively well-defined string-localized correlation functions to charged "particles" without their infinite infrared clouds of photons blurring the large time asymptotic picture; "this century problem" (dating back to Bloch-Nordsiek) seems to have a conceptually quite demanding natural solution [37].

[28]Pointlike matter-fields with a bad high energy behavior already appeared in the "unitary gauge" of the oldest results about massive QED [66].

[29]In a certain sense the zero mass analog of the Proca field is the (noncovariant) Coulomb field-

[30]The infrared-finiteness of correlations in the string-localized Hilbert space description shows that the infrared divergences in the standard approach are a result of the of the nonexistence of physical pointlike fields in the massless limit.

In this aspect the stringlike Hilbert space formulation is superior to the Krein space formulation. It presents for the first time a *rigorous perturbative way* to check the asymptotic freedom statements based on the mass-independent beta function within a formulation with perturbative well-defined Callen-Symanzik equation. In general the pointlike fields which appear in the adiabatic equivalence relation of massive vectormeson models constitute a perturbative construct since the pointlike Hilbert space formalism is not renormalizable. In fact they have a good chance to be SLF (strictly localizable) fields in the sense of Jaffe [62] which, although not being Schwartz distributions as a result of their bad short distance properties, are still localizable. The pointlike Hilbert space field $\hat{\psi}$ in (17) is connected with its renormalizable BRST or string-localized counterpart by an operator gauge transformation in terms of an exponential in the Stückelberg field which makes the physical spinor field nonrenormalizable but maintains its affiliation to a finite-parametric QFT[31].

The disadvantage of fighting the weakening of localization by using Krein spaces instead of using string-localized fields in Hilbert space shows up most forcefully when higher spin zero mass fields participate in the interaction. In that case pointlike objects which are analogous to Proca fields simply do not exist; the charged matter fields of QED cannot be pointlike generated. In other words pointlike physical descriptions of charged fields in QED in the Krein space setting do not exist. There is also the problem of the physical credibility of the result that the consistency of the renormalization of massive Y-M in the Krein setting requires the presence of scalar multiplets which is the observation on which the vernacular "(either) Higgs particles or death (of QFT)" (i.e. the yet unproven statement that massive Y-M interactions without the presence of neutral massive scalar fields is inconsistent with the principles of QFT) is based. The Krein space formulation (BRST formalism) is only consistent with a linear transformation of the Proca field [65]. The nonlinear transformation (15) which seems to follow from the adiabatic equivalence principle leads to the presence of additional second order terms which may still lead to a different conclusion. The claim that the Higgs particle creates the masses (including its own) of all quantum matter as a result of formal manipulations of Yukawa couplings is a the result of confusing formal Lagrangian field manipulations with intrinsic properties of a model of QFT[32]. It may also be a public relation trick (the "God particle") to sell the by for most expensive experiment to the public or the result of a mixture of both.

The Krein method did however confirm the veracity of Stora's statement that the Lagrangian structure of gauge theory is not the result of a group theoretic imposition of symmetries but rather a consequence of the renormalization requirement [65], a result which continues to hold in the Hilbert space formalism for string-localized fields.

Stringlike localization also entered the axiomatic approach to theories with mass-gaps as the *most general localization of charge-carrying fields associated with pointlike generated observables which can be derived from the mass-gap assumption*; this is the result of a deep structural theorem by Fredenhagen and Buchholz [3]. It seems likely that the strings of matter fields in massive gauge theories (which unlike the vectormeson

[31] Exponentials of free scalar fields are SLF [62] and it is believed that this is true in general for exponentials of fields with scale dimension d=1.

[32] The proponents of these terminology forgot to explain how, by looking at physical correlations in QFT, they can distinguish between a own from a given mass.

strings cannot be removed by passing to field strength by differentiation) are generators of Buchholz-Fredenhagen spacelike-cone-localized operators. In this case the massive higher spin strings would be concrete realizations of the somewhat abstract structural B-F theorem. They may even have pointlike representatives in their Borchers class of relative local fields which are not of the Wightman type.

## 3.2 Remarks on modular localization

There remains the problem of what this significant enlargement of renormalizability and localization means in terms of its physical consequences. We will return to this problem in the next subsection, after having explained some ideas about modular localization in the simple context of Wigner representations and their relation to the operator-algebraic formulation of modular localization.

It has been realized, first in a special context in [69], and afterwards in a general rigorous setting in [19] (see also [67][20]), that there exists a *natural localization structure* on the Wigner representation space for any positive energy representation of the proper Poincaré group. A convenient presentation can be given in the context of spinless particle for which the $(m > 0, s = 0)$ Wigner one-particle space is the Hilbert space $H_1$ of (momentum space) wave functions with the inner product

$$H_1 : (\varphi_1, \varphi_2) = \int \bar{\varphi}_1(p)\varphi_2(p)\frac{d^3p}{2p_0}, \quad \hat{\varphi}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int e^{-ipx}\varphi(p)\frac{d^3p}{2p_0} \tag{18}$$

$$g \in \mathcal{P}_+ = \mathcal{P}_+^\uparrow \cup \mathcal{P}_+^\downarrow \quad U(g)(H_1^{(1)} \oplus H_1^{(2)}) = \begin{cases} U(g)H_1^{(1)}, \ g \in \mathcal{P}_+^\uparrow \\ U^{anti}(g)H_1^{(1)}, \ g \in \mathcal{P}_+^\downarrow \end{cases}$$

In this case the covariant x-space amplitude is simply the on-shell Fourier transform of this wave function whereas for $(m \geq 0; s \geq 1/2)$ the covariant spacetime wave function is more involved as a consequence of the presence of intertwiners $u(p,s)$ between the manifestly unitary and the covariant form of the representation [56]. The second line expresses the action of the proper part of the Poincaré group $\mathcal{P}_+$ which includes all $det(g) = 1$ transformations; it consists of the action of the connected part on the irreducible Wigner representation space $H_1$ and the action of a time-reversing antiunitary action on a second copy of $H_1$ (whose wave functions refer to antiparticles which reduce to particles in the charge-neutral case).

Selecting a wedge region e.g. $W_0 = \{x \in \mathbb{R}^d, x^{d-1} > |x^0|\}$, one notices that the unitary wedge-preserving boost $U(\Lambda_W(\chi = -2\pi t)) =: \Delta^{it}$ commutes with the antiunitary reflection $J_W$ on the edge of the e.g. t-z wedge[33] $(x_0 \rightarrow -x_0, z \rightarrow -z; \ \vec{x}_{transvere} \ fixed)$. This has the unusual (and perhaps even unexpected) consequence that the unbounded and antilinear operator

$$S_W := J_W\Delta^{\frac{1}{2}}, \quad S_W^2 \subset 1 \tag{19}$$
$$since \quad J\Delta^{\frac{1}{2}}J = \Delta^{-\frac{1}{2}}$$

[33] Wedges in general position are obtained from the t-z wedge by Poincaré transformations.

which is intrinsically defined in terms of Wigner representation data, is *involutive on its dense domain* and therefore has a unique closure with $ranS = domS$ (unchanged notation for the closure).

The involutivity means that the $S$-operator has $\pm 1$ eigenspaces; since it is antilinear, the +space multiplied with $i$ changes the sign and becomes the -space; hence it suffices to introduce a notation for just one real eigenspace

$$\begin{aligned} & K(W) = \{domain\ of\ \Delta_W^{\frac{1}{2}},\ S_W\psi = \psi\} \\ & J_W K(W) = K(W') = K(W)',\ duality \\ & \overline{K(W) + iK(W)} = H_1,\ K(W) \cap iK(W) = 0 \end{aligned} \tag{20}$$

It is important to be aware that one is dealing here with *real* (closed) subspaces $K$ of the complex one-particle Wigner representation space $H_1$. An alternative is to directly work with the complex dense subspaces $K(W) + iK(W)$ as in the third line. Introducing the *graph norm* in terms of the positive operator $\Delta$, the dense complex subspace becomes a Hilbert space $H_{1,\Delta}$ in its own right. The upper dash on regions denotes the causal disjoint (the opposite wedge), whereas the dash on real subspaces means the symplectic complement with respect to the symplectic form $Im(\cdot,\cdot)$ on $H$. All the definition work for arbitrary positive energy representations of the Poincaré group [19].

The two properties in the third line are the defining relations of what is called the *standardness property* of a real subspace[34]; any abstract standard subspace K of an arbitrary real Hilbert with a $K$-operator space permits to define an abstract $S$-operator in its complexified Hilbert space

$$\begin{aligned} & S(\psi + i\varphi) = \psi - i\varphi,\ S = J\Delta^{\frac{1}{2}} \\ & domS = dom\Delta^{\frac{1}{2}} = K + iK \end{aligned} \tag{21}$$

whose polar decomposition (written in the second line) yields two modular objects, a unitary modular group $\Delta^{it}$ and an antiunitary reflection which generally have however no geometric interpretation in terms of localization. The domain of the Tomita $S$-operator is the same as the domain of $\Delta^{\frac{1}{2}}$, namely the real sum of the $K$ space and its imaginary multiple. Note that for the physical case at hand, this domain is intrinsically determined solely in terms of the Wigner group representation theory, showing the close relation between localization and covariance.

The $K$-spaces are the real parts of these complex $domS$, and in contrast to the complex domain spaces they are closed as real subspaces of the Hilbert space (corresponding to the one-particle projection of the real subspaces generated by Hermitian Segal field operators). Their symplectic complement can be written in terms of the action of the $J$ operator and leads to the K-space of the causal disjoint wedge $W'$ (Haag duality)

$$K_W' := \{\chi |\ Im(\chi,\varphi) = 0, all\ \varphi \in K_W\} = J_W K_W = K_{W'} \tag{22}$$

[34] According to the Reeh-Schlieder theorem a local algebra $\mathcal{A}(\mathcal{O})$ in QFT is in standard position with respect to the vacuum i.e. it acts on the vacuum in a cyclic and separating manner. The spatial standardness, which follows directly from Wigner representation theory, is just the one-particle projection of the Reeh-Schlieder property.

The extension of W-localization to arbitrary spacetime regions $\mathcal{O}$ is done by representing the causal closure $\mathcal{O}''$ as an intersection of wedges and defining $K_{\mathcal{O}}$ as the corresponding intersection of wedge spaces

$$K_{\mathcal{O}} = K_{\mathcal{O}''} \equiv \bigcap_{W \supset \mathcal{O}''} K_W, \quad \mathcal{O}'' = \textit{causal completion of } \mathcal{O} \tag{23}$$

These K-spaces lead via (21) and (23) to the modular operators associated with $K_{\mathcal{O}}$.

For those who are familiar with Weinberg's intertwiner formalism [56] relating the $(m,s)$ Wigner representation to the dotted/undotted spinor formalism, it may be helpful to recall the resulting "master formula"

$$\begin{aligned} \Psi^{(A,\dot{B})}(x) &= \frac{1}{(2\pi)^{\frac{3}{2}}} \int (e^{-ipx} \sum_{s_3=\pm s} u^{(A,\dot{B})}(p,s_3) a(p,s_3) + \\ &\quad + e^{ipx} \sum_{s_3=\pm s} v^{(A,\dot{B})}(p,s_3) b^*(p,s_3)) \frac{d^3p}{2\omega} \\ \sum_{s_3=\pm s} & u^{(A,\dot{B})}(p,s_3) a(p,s_3) \to u(p,e) \cdot a(p) \end{aligned} \tag{24}$$

where the $a,b$ amplitudes correspond to the Wigner momentum space wave functions of particles/antiparticles and the $u,v$ represent the intertwiner and its charge conjugate. For the third class (infinite spin, last line), the sum over spin components has to be replaced by an inner product between a $p,e$-dependent infinite component intertwiner $u$ and an infinite component $a(p)$, since in this case Wigner's "little space" is infinite dimensional. The $\Psi(x)$ respectively $\Psi(x,e)$ are "generating wave functions" i.e. they are wavefunction-valued Schwartz distributions which by smearing with $\mathcal{O}$-supported test functions become $\mathcal{O}$-localized wave functions. Adding the opposite frequency antiparticle contribution one obtains the above formula which, by re-interpreting the $a^{\#}, b^{\#}$ as creation/annihilation operators (second quantization functor), describes point-respectively stringlike free fields. The resulting operator-valued Schwartz distributions are "global" generators in the sense that they generate $\mathcal{O}$-localized operators $\Psi(f)$ for *all* $\mathcal{O}$ by "smearing" them with $\mathcal{O}$-supported test functions $suppf \in \mathcal{O}$.

Only in the massive case the full spectrum of spinorial indices $A, \dot{B}$ is exhausted (9), whereas the massless case leads to restrictions (10) which come about because pointlike "field-strength" are allowed, whereas pointlike "potentials" are rejected (related to the different zero mass little group). This awareness about the conceptual clash between localization and the Hilbert space[35] is important for the introduction of string-localization.

Whereas Weinberg [56] uses (the computational somewhat easier manageable) covariance requirement[36], the modular localization method is based on the direct construction of localized Wigner subspaces and their stringlike generators. In that case the intertwiners

[35] In the case of [20] this awareness came from the prior use of "modular localization" starting in [68][69] but foremost (covering *all* positive energy Wigner representations) in [19].

[36] For wave functions and free fields covariance is synonymous with causal localization, but in the presence of interaction the localization of operators and that of states split apart.

depend on the spacelike direction $e$ which is not a parameter but, similar to the localization point, a variable in terms of which the field fluctuates [20] and whose presence allows the short distance fluctuations in $x$ to be milder than in case of pointlike fields.

The short-distance reducing property of the generating stringlike fields is indispensable in the implementation of renormalizable perturbation theory in Hilbert space for interactions involving spins $s > 1/2$[37]. Whereas pointlike fields are the mediators between classical and quantum localization, the stringlike fields are outside the Lagrangian or functional quantization setting since they are not solutions of Euler-Lagrange equations; enforcing the latter one arrives at pointlike fields in Krein space. String-localization lowers the power-counting limit, but renders the application of the iterative Epstein-Glaser machinery [12] more involved. In the next section it will be shown that modular localization is essential for *generalizing Wigner's intrinsic representation theoretical approach to the* (non-perturbative) *realm of interacting localized observable algebras.*

In order to arrive at Haag's algebraic setting of local quantum physics in the absence of interactions, one may avoid "field coordinatizations" and apply the Weyl functor $\Gamma$ (or its fermionic counterpart) directly to *wave function subspaces* where upon they are functorially passed directly to operator algebras, symbolically indicated by the functorial relation

$$K_{\mathcal{O}} \stackrel{\Gamma}{\rightarrow} \mathcal{A}(\mathcal{O}) \tag{25}$$

The functorial map $\Gamma$ *also* relates the modular operators $S, J, \Delta$ from the Wigner wave function setting directly with their "second quantized" counterparts $S_{Fock}, J_{Fock}, \Delta_{Fock}$ in Wigner-Fock space; it is then straightforward to check that they are precisely the modular operators of the Tomita-Takesaki modular theory applied to causally localized operator algebras (using from now on the shorter $S, J, \Delta$ notation for modular objects in operator algebras).

$$\begin{aligned} \sigma_t(\mathcal{A}(\mathcal{O})) &\equiv \Delta^{it}\mathcal{A}(\mathcal{O})\Delta^{-it} = \mathcal{A}(\mathcal{O}) \\ J\mathcal{A}(\mathcal{O})J &= \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}') \end{aligned} \tag{26}$$

In the absence of interactions these operator relation are consequences of the modular relations for Wigner representations. The *Tomita-Takesaki theory secures their general existence for standard pairs* $(A, \Omega)$ i.e. an operator algebras $\mathcal{A}$ and a state vector $\Omega \in H$ on which $\mathcal{A}$ acts cyclic and separating (no annihilators of $\Omega$ in $\mathcal{A}$). The polar decomposition of the antilinear closed Tomita $S$-operator leads to the unitary modular automorphism group $\Delta^{it}$ associated with the subalgebra $\mathcal{A}(\mathcal{O}) \subset B(H)$ and the vacuum state vector $\Omega$ i.e. with the pair $(\mathcal{A}(\mathcal{O}), \Omega)$.

Although $B(H)$ is generated from the two commuting algebras $\mathcal{A}(\mathcal{O})$ and $\mathcal{A}(\mathcal{O})'$, they do not form a tensor product in $B(H)$; hence the standard quantum-information and QM concepts concerning entanglement and density matrices are not applicable; the QFT realization of entanglement for monads is stronger[38]. As a result of this "monad entanglement", the impure state results from just restricting the vacuum to the monad,

[37] These are also prcisely those interactions in which the absence of mass gaps does not lead to problems with the particle structure.

[38] The localization entropy of the vacuum entanglement for $\mathcal{A}(\mathcal{O})/\mathcal{A}(\mathcal{O}\prime)$ is infinite.

one does not have to average over degrees of freedom in order to convert entangled states into density matrices, as it is necessary in the standard quantum information situation where instead of a monad one has a $B(H)$ type algebra associated with a factor space $H$.

As mentioned modular localization of operators is more restrictive than modular localization of states. Outside of perturbation theory it is perfectly conceivable that a state vector generated by applying an *algebraically indecomposable* stringlike localized field to the vacuum is decomposable into a direct sum/integral over pointlike generated Wigner representations; in fact all positive energy representations which do not contain components to the infinite spin representations allow such continuous decomposition. An important illustration of this fact is provided by the charge-carrying infraparticle fields in QED.

The only case for which the modular localization theory (the adaptation of the Tomita-Takesaki modular theory to the causal localization principle of QFT) has a geometric interpretation, independent of whether interactions are present or not and independent of the type of quantum matter, is the wedge region i.e. the Poincaré transforms of the standard wedge $W = \{|x_0| < x_3|\mathbf{x}_{tr} \in \mathbb{R}^2\}$. In that case the modular group is the wedge-preserving Lorentz boost, and the $J$ represents a reflection on the edge of the wedge, i.e. it is up to a $\pi$-rotation equal to the antiunitary TCP operator. The TCP invariance as derived by Jost [42], combined with scattering theory (the TCP transformation of the S-matrix) leads to the relation

$$J = S_{scat} J_{in} \tag{27}$$

which in [68][69] has been applied to constructive problems of integrable QFTs. This is a relation which goes much beyond scattering theory; in fact it only holds in local quantum physics since it attributes the new role of a relative modular invariant of causal localization to the S-matrix which has no counterpart in QM.

This opens an unexpected possibility of a new access to QFT, in which the first step is the construction of generators for the wedge-localized algebra $\mathcal{A}(W)$ with the aim to obtain spacelike cone-localized (with strings as a core) or double cone-localized algebras (with a point as core) from *intersecting wedge algebras.* In this top-to-bottom approach, which is based on the intuitive idea that the larger the localization region, the better the chance to construct generators with milder vacuum polarization, pointlike fields would only appear at the end of the construction. In fact according to the underlying philosophy that all relevant physical data can be obtained from localized algebras, the use of individual operators within such an algebra may be avoided altogether; the *relative positioning of the localized algebras should account for all physical phenomena in particle physics.* The next section presents the first step in such a construction.

The only prerequisites for the general (abstract) case is the "standardness" of the pair $(\mathcal{A}, \Omega)$, where "standard" in the theory of operator algebras means that $\Omega$ is a cyclic and separating vector with respect to $\mathcal{A}$, a property *which in QFT is always fulfilled* for localized $\mathcal{A}(\mathcal{O})'s$ (thanks to the validity of the Reeh-Schlieder theorem [3]). These local operator algebras of QFT are what has been in previous publications referred as *monads* [10]*;* as mentioned before, their properties are remarkably different from the algebra of all bounded operators $B(H)$ which one encounters for Born-localized algebras in QM [11]. For general localization regions the one-parametric modular unitaries have no geometric interpretation (they describe a kind of fuzzy action inside $\mathcal{O}$), but they are uniquely determined in terms of intersections of their geometric $W$-counterparts and are expected to become important in any top-to-bottom construction of models of QFT. Even in the simpler context of localized subspaces $K_{\mathcal{O}}$ related to Wigner's positive energy representation theory for the Poincaré group and its functorial relation to free fields, these concepts have shown to be useful [19].

The most important conceptual contribution of modular localization theory in the context of the present work is the assertion that the reduction of the global vacuum (and also finite energy particle states) to a local operator algebra $\mathcal{A}(\mathcal{O})$ leads to a "thermal" KMS state for which the "thermal Hamiltonian" $H_{mod}$ is the generator of the modular unitary group

$$e^{-i\tau H_{mod}} := \Delta^{i\tau} \tag{28}$$
$$\langle AB\rangle = \left\langle Be^{-H_{mod}}A\right\rangle$$

where the second line has the same KMS form as in the case of a heat bath thermal systems (after rewriting the Gibbs trace formula into the state-setting of the *open system formulation* of statistical mechanics[39] [3]). Whereas the trace formulation breaks down in the thermodynamic limit, this analytic KMS formula (asserting analyticity in $-1 < Im\tau < 0$) remains. It is in this and only in this thermodynamic limit, that a monad algebra also appears in QM.

As mentioned in the introduction, the intrinsic thermal aspect of localization is the reason why the probability issue in QFT is conceptually different from the Born probability in QM.

Closely related to the modular localization is the "GPS characterization" of a QFT (including its Poincaré spacetime symmetry, as well as the internal symmetries of its quantum matter content) in *terms of modular positioning of a finite number of monads* in a shared Hilbert space. For d=1+1 chiral models the minimal number of copies is 2, whereas in d=1+3 the smallest number for a GPS construction is 7 [70]. This way of looking at QFT is an *extreme relational point of view* in terms of objects which have no internal structure by themselves; this explains the terminology "monad" (Leibnitz's point of view about reality, but now in the context of quantum matter) [70][11]. This view of QFT exposes its radically holistic structure in the most forceful way. In praxis one starts with one monad and assumes that one knows the action of the Poincaré group on it [68][69]. This generates a net of transformed monads which by forming intersections lead to monads associated to smaller regions (spacelike cones or double cones). This was precisely the way in which the existence of factorizing models was shown [23], where the nontriviality of the intersection was established by verifying the "nuclearity property" of degrees of freedom.

In order to show the power of this new viewpoint for particle physics, the following last subsection of this section shows some consequences of string localization which are relevant for Standard Model physics.

[39] Ground state problems in QM do not come anywhere near such a tight situation.

## 3.3 Expected consequences for Standard Model physics

Since its inception the "Higgs mechanism" has been the cause of many conceptual misunderstandings [71]. In the 70s there were two mechanisms about how to couple neutral scalar fields to pointlike massive vektormesons. One was in terms of spontaneous symmetry breaking in which the zero mass Goldstone degree of freedom was converted into an additional degree of freedom which changed the photon with two helicity degrees of freedom into a massive vectormeson with three polarization degrees of freedom, the famous Higgs mechanism. This mechanism is metaphoric, it is a "pons asini" to suggest that an interaction of a neutral scalar with vectormeson is somehow "more renormalizable" than massive scalar QED or other couplings of vectormesons to matter fields or among themselves. The idea was that the indefinite metric setting known from QED and the renormalization preserving Goldstone spontaneous symmetry breaking create a situation which distinguishes the massive neutral case from its massive charged counterparts. It will be shown in the sequel that there exists no distinction with respect to renormalizability between these different models; QFT does not permit a distinction between an elementary and an "induced" mass of vectormesons and all these couplings are equally nonrenormalizable in the sense of pointlike interactions.

The other mechanism which also was expected to overcome the hurdle of the power-counting limit of renormalizability was Schwinger's "screened" electromagnetism [72] for which Swieca [73][71] found a rigorous formulation (see also [51]). It applies to models of QFT which possess identically conserved currents (Maxwell-type currents). The difference between screening currents and spontaneous symmetry currents lies in their long distance behavior:

$$
\begin{aligned}
&screening:\ Q=\int j_0(x)d^3x=0,\ \partial^\mu j_\mu=0 \qquad (29)\\
&spont.symm.-breaking:\int j_0(x)d^3x=\infty
\end{aligned}
$$

Obviously the analogy to the quantum mechanical Debeye screening led to this idea. In that case long range potentials between electrically charged particles in a medium in which both $\pm$ charges are present become effectively short ranged. But whereas in QM this is an effective mechanism which does not alter the fundamental quantum mechanical structure, screening in QFT is more radical. The conceptually different structure of causal QED requires a change of particle spectrum from photons to massive vectormesons and a change of the charged to neutral matter; hence such analogies have to be taken with a grain of salt.

Nevertheless the following screening theorem holds for all known couplings of vectormesons. It does not distinguish between elementary and induced mass generation of vectormesons for a good reason: there is no intrinsic distinction within QFT. It is instructive to look at Swieca's theorem and its consequences in more detail.

**Theorem 2** ***(Swieca [73]) In gauge theories[40] with a mass gap the Maxwell charge is screened***

$$mass\ gap\ and\ \partial^{\nu}F_{\mu\nu}=j_{\mu}\ \curvearrowright Q=\text{"}\int d^{3}xj_{0}(x)\text{"}=0 \tag{30}$$

**It is somewhat surprising that the more physical screening picture for massive vectormesons did not take hold in the 70s [71] even though there were some who noticed that there was a fruitful clash between the screening and the broken symmetry picture [74]. Only nowadays we understand clearly that the Higgs mechanism which attributes a special role with respect to renormalizability to couplings of massive vectormesons and neutral as opposed to charged matter is fictitious and that the shared screening picture for good reasons does not distinguish between the nature of mass of vectormesons in their coupling to matter.**

**Again this underlines the holistic nature of QFT as compared to QM. Whereas QM is ambivalent with respect to interpretations (viz. the insensitivity against changing the spacetime dimensionality, the treatment of Coulomb interactions and their Debeye-sceened version in the same mathematical setting), QFT punishes wrong interpretation as the claim that massive vectormesons can only arise through the Higgs mechanism i.e. the claim that interacting massive vectormesons need neutral scalar companions. All these different models involving massive vectormesons share one property: they are realizations of the new adiabatic equivalence principle: the (formally) nonrenormalizable interactions with Proca vectormesons and their renormalizable counterparts in the stringlike setting preserve their free field property of being relative local operator-valued distributions (same Borchers class) in the presence of interactions. Different from Krein-space treatments, one never has to leave the Hilbert space setting of QT. It is the class of relatively localized (composite) covariant fields, which includes pointlike as well as stringlike fields, which determines the physical content, the different fields are only different singular coordinatizations.**

**This begs the question of the mathematical status of pointlike matter fields in theories which obey the adiabatic equivalence principle, concretely the status of $A_{\mu}^{P}(x)$ and $\Psi(x)$ as compared to the renormalizable $A_{\mu}(x,e)$ and $\Psi(x,e)$. The subtlety of the problem justifies to repeat statements which were already made in subsection 3.1 in a different formulation. There is as yet no final answer to this question, but the obvious conjecture is that the stringlike vectormeson can be changed into a equivalent pointlike field strength (or a composite), whereas no linear operation on stringlike matter fields is able to undo the string localization; the exponential nature of the "operator gauge transformation" which implements the adiabatic equivalence suggests that the pointlike matter fields are not Wightman fields (not bounded by p-space**

[40] In the string-localized formulation of Y-M models the string-localized color-current resulting form the derivative of the color field strength is identically conserved.

polynomials) but rather nontemperate SLF (strictly localizable fields) as first defined in the context of QFT by Arthur Jaffe[41] [62][61]. It is presently not clear whether the equivalence between the pointlike and Haag's LQP formulation also extends to situations which do not admit the full Schwartz class of localizable test functions.

This conceptual and mathematical new setting has consequences for gauge theories. For the first time in their history one now has an access which is intrinsic i.e. independent of quantization i.e. does not rely on the quantization formalism which requires to resolve the localization/Hilbert clash in terms of pointlike vectorpotentials in Krein space but rather permits to take the more physical route in terms of stringlike localization in Hilbert space which is suggested by the intrinsic Wigner representation theory and which also incorporates matter fields which in the BRST setting only exist as pointlike chimeras (ghosts). In classical field theory vectorpotentials are fields as other fields, whereas quantization attributes to them properties which are only consistent with Krein spaces: only the gauge invariant objects are physical in the sense of consistency with the positivity of QT and matter fields are not among the physical quantities. The substitution of this gauge formalism by stringlike localization does not only follow the philosophy of Occam's razor, but it also attributes a physical reality to the coupled quantum matter and also explains why it cannot be described in terms of pointlike localized Wightman fields.

Even though the scalar pointlike field $h(x)$ appearing in the relation

$$d_e A_\mu(x,e) = d_e \partial_\mu \phi(x,e) =: \partial_\mu h(x) \tag{31}$$

resembles a Higgs field, the new setting has no relation to the Higgs mechanism. In particular no symmetry is spontaneously broken in fact it has never been clear to the author which symmetry was it which was termed as "broken". The only symmetry which is broken in the case of a real scalar field coupled to a complex one is the $Z_2$ symmetry but since for the coupling of a scalar field there is no principle which rules out the appearance of odd terms in the interaction, such a symmetry was not there from the start. It is interesting to confront these new findings with the present discussions around the "discovery of the Higgs boson".

The before mentioned "Higgs or death (of QFT)" situation which is the outcome of almost 40 years of stagnation about the Standard Model is very unhealthy for experimental physics. Without theoretical alternatives experimental physicists are under enormous pressure to present the result of their findings as a confirmation of what the overwhelming majority of theoreticians tell them (and to justify the enormous costs of such experiments). But this also applies to the opposite direction, theoreticians tend to consider an issue as closed after it obtained its experimental verification. I am pleading here for an unbiased investigation. QFT produces perfectly renormalizable interactions in the string-localized setting in which the only pointlike object

[41] Jaffe also showed that exponentials of scalar free fields belong to this class.

**is the scalar selfadjoint field $h$ (31) which is uniquely associated with a string-localized vector potential and has no new degrees of freedom of its own. This is not only an alternative to the Higgs mechanism but it also does not share its metaphoric nature and requires no application of Occam's razor. It also liberates experimentalists from relating their important findings with doubtful theoretical scenarios.**

**Apart from the problem of a construction of physical matter fields the BRST treatment of massive vectormesons leads to identical gauge invariant ($e$independent) in results. This is not the case in for self-coupled massive Y-M vectormesons. The BRST formalism in which the return to the Hilbert space is achieved in terms of a ghost charge formalism is only on par with the string setting for the gauge invariant observables in case of abelian self-couplings of vectormesons; the principle of adiabatic equivalences in Hilbert space requires nonlinear contributions resulting from operator gauge transformations[42] on the Proca field in terms of Stückelberg scalars which have no counterpart in the Krein setting [65][75]. Therefore there remain doubts about the validity of that setting and its results which only can be settled by doing the second order calculations in the stringlike setting (in progress).**

**The pointlike setting which is already best by short distance problems (nontemperedness of fields as short distances) develops additional infrared problems which can only be controlled by staying with the renormalizable stringlike formulation. The impossibility of defining correlation functions for massless Y-M interactions has misled particle theorists to claim that "long distances are nonperturbative". The correct interpretation would be to say that pointlike fields (except certain composites) are ill-defined but stringlike objects remain as well-defined as pointlike objects in low spin renormalizable interactions $s \leq 1/2$ which despite zero mass have neither off-shell nor on-shell infrared problems.**

**Any zero mass gauge theory, whether described in terms of string- or point-like localized fields, has of course those infraparticle particle problems which already led Bloch and Nordsiek in the 30s to modified description which they discussed in terms of a non-covariant model. Later more elegant entirely field theoretic descriptions by Yennie, Frautschi and Suura incorporated these ideas into the standard formulation of QFT by avoiding the infrared divergent scattering amplitudes[43] in terms of photon inclusive cross sections. Also this treatment, which works with (noncovariant) infrared cutoffs, does not represent a foundational solution for infraparticle problems without mass shell singularities in correlation functions, but it at least gives a good agreement with the observational facts. Recently a new attempt at a foundational understanding of the infraparticle problem was initiated in [80][81] and in which the noncompact spatial extensions of infinite clouds which comes with the**

---

[42] We remind the reader that the quantum counterpart of classical gauge transformation only enters through the implementation of the superordinate principle of adiabatic equivalence.

[43] In the nonperturbative LSZ scattering theory these amplitudes vanish as a result of the softer than mass-shell delta functions singularities of these "infraparticles".

**semiinfinite spacelike strings (the $e$-dependence) is avoided by restricting the localization of the whole theory to the (any) forward light cone which leads to a natural loss of infinite photon clouds without the necessity of a nonintrinsic photon inclusiveness.**

**In the nonabelian case the contributions of the strings in causal crossing positions (i.e. outside there spacelike separations) but for $x \neq x'$ are stronger which may account for the facts that the infrared divergencies in the pointlike BRST description already occurs in the off-shell correlations before the issue of the mass-shell behavior arises. In order to support such an idea it is helpful to revisit the infinite spin representation [19][20] from which the whole idea of string localization started. Whereas for massless finite helicity objects the application of stringlike operators cannot be directly seen in the states which result from their application to the vacuum (which remain completely reducible in terms of sums and direct integrals over pointlike generated Wigner representations), the physical consequences of "irreducible" [44] are extremely drastic. The idealization of a counter in terms of a particular kind of compact spacetime localized (or at least "quasilocal" observable [3]) makes it impossible for an irreducible string state to activate such a counter. It is also questionable whether such matter can interact with normal matter. If an interaction is possible at all it is very different from the case of the above interaction of string field. This would be the ideal candidate for dark matter if it would not be for the idea that black matter is massive (assumed in all models, but apparently not established by astrophysical observations).**

**If the interaction between massless string-localized vectorpotentials and with massive matter fields is "switched on" (apologies for the use of a metaphoric picture) the string-localization of the interacting vectorpotentials becomes stronger and the original pointlike massive matter will be strongly "stringlike contaminated". But unless the interaction produces also a components of the rather large class of infinite spin representation (improbable, but not excluded by known theorems), it ends up being string-localized in an algebraic sense which however does not affect the resolution of states in terms of pointlike generated "counter responsive" states.**

**It would be extremely interesting to calculate the beta function of massless beta function and see whether it agrees with the famous Politzer-Gross-Wilczek dimensional regularization but without having established a Callan-Symanzik equation for correlation functions which is the conceptual home of a beta function. One can either do this in the string-localized massive version appealing to the property of mass independence of $\beta(g)$ or go directly to the perturbatively well-defined string-localized correlation functions. The calculation may be easier in the BRST setting but, as a result of the nonexistence of a pointlike massless physical limit it is also less credible.**

**The reader may wonder why the word "supersymmetry" occurred in this paper only in connection with the solution of the Majorana's mathematical**

[44] This means that such states cannot be resolved in terms of integrals over pointlike generated Wigner wave functions.

**problem (the infinite component "superstring representations" of the Poincaré group) and not with particle physics. There is a simple answer; whereas the main physical motivation for supersymmetry (namely the improvement of short distance properties in order to increase the range of renormalizability, turned out to be an illusion), the use of string-localized fields in Hilbert space really adds to the finite number of pointlike renormalizable couplings infinitely many renormalizable stringlike interactions for higher spin field. Of course not all of them possess compactly localized observable subalgebras which is the prerequisite for the validity of the adiabatic equivalence requirement.**

# 4 Generators of wedge algebras, extension of Wigner representation theory in the presence of interactions

Theoretical physics is one of the few areas of human endeavor in which the identification of an error may be as important as the discovery of a new theory. This is especially the case if the committed error is related to a lack of understanding or misunderstanding of the causal localization principle which is the basis of QFT as the first S-matrix attempt in the 60s which tried to use analytic on-shell crossing properties without the knowledge how such analytic properties arise from the off-shell localization properties. Whereas off-shell analytic properties of correlation function were systematically analyzed in the pathbreaking work of Bargmann, Hall and Wightman [27], it was already clear at the time of the dispersion relations that on-shell analytic properties are of a different conceptual caliber, and the field-particle relation coming from LSZ scattering theory is not sufficient for for their understanding. In some special cases of elastic scattering the application of the intricate mathematics of several complex variables and the formation of natural analytic extensions [14] led to a proof of the crossing analyticity in special cases. But the derivation did not reveal what we know nowadays, namely that the particle crossing identity is closely related and in fact can be derived from the KMS identity of modular wedge localization. The main difference to the Unruh effect is that one has to convert field states in the presence of interactions into particle states, but this again can be achieved in terms of modular localization.

Only after the arrival of modular localization and its role in the construction of d=1+1 integrable models for the spacetime identification of the Zamolodchikov algebra structure the understanding began to improve. The crucial step was the realization that the S-matrix was not only an operator resulting from time-dependent scattering theory (which it is in every QT), but also a relative modular invariant of wedge-localized algebras. This led to the idea that the crossing property and its analytic aspects in terms of particle rapidities is a result of a particle translation of the analytic KMS identity for operators localized in the wedge for which the analyticity refers to the hyperbolic angle of the wedge-preserving Lorentz transformation. The derivation of the crossing relation from the same modular localization principle which solves the E-J conundrum and explains the KMS temperature of the Unruh and Hawking effect is surprising; this and a closely related proposal for a general on-shell construction [10] (which extends the successful construction of integrable models from the structure of their generators of wedge algebras [23]) is the theme of this section.

In this way the original aim of Mandelstam's on-shell project for finding a route to particle theory, which is different to quantization and perturbation theory and stays closer to directly observational accessible objects is recovered, and the picture puzzle trap of ST (section 2) which led to wrong understandings of crossing is avoided. It is closely related to the top-to-bottom oriented LQP setting in which the desired concepts are laid down before their mathematical and computational implementation starts; this is opposite to quantization approaches were the properties of objects and their interpretation come to light only after having done the calculations following the quantization rules. Besides aspects which are accessible by quantization (section 3), there are also properties which cannot be understood in this way as the E-J conundrum or other thermal[45] aspects of modular localization as the Unruh and Hawking effects as well as localization-caused entropy which characterize the modular statistical mechanics aspect of localized ensembles. This section adds the particle crossing and the closely related on-shell construction method to these properties whose understanding requires the use of the modular localization principle.

In retrospect it is clear why Mandelstam's project had no chance to succeed in the 60s and 70s; the necessary conceptual tools were not available at a time in which the impressive success of renormalized perturbation was still on peoples mind and QFT was simply that theory behind Lagrangian/functional quantization.

The most difficult aspect of modular localization is the comprehension of the big separation it creates between particles and fields in the presence of interactions. Whereas these two concepts are closely related in a functorial way in the absence of interactions, jn the presence of interactions they drift apart in such a way that it takes great conceptual efforts to understand what is left. The efford goes significantly beyond the use of modular localization needed for the E-J conundrum and Unruh-Hawking effects. These efforts start with observation that the S-matrix is not only that object resulting from the well-understood relation between the large time asymptotic behavior of fields with particles (which its shares with QM), but in modular localizable theories as QFT, it is also a relative modular invariant associated with the structure of an interacting wedge algebra relative to its free counterpart (generated by incoming fields). The important point is that this property where fields and particles are brought together is not asymptotic, it rather contains a noncompact form of localization.

In order to motivate the reader to enter a journey which takes him far away from text-book QFT, it is helpful with a theorem which shows that the familiar particle-field relations breaks down in the presence of *any* interaction. The following theorem shows that the separation between particles and *interacting* localized fields and their algebras is very drastic indeed [10]:

**Theorem 3** *(Mund's algebraic extension [76] of the old J-S theorem [27]) A Poincaré-covariant QFT in* $d \geq 1+2$ *fulfilling the mass-gap hypothesis and containing a sufficiently*

[45] Here thermal is not necessarily referring to what can be measured with a thermometer [6] but rather characterizes the specific (modular) impurity which results from a $\mathcal{A}(\mathcal{O})$-restriced vacuum.

*large set of "temperate" wedge-like localized vacuum polarization-free one-particle generators (PFGs) is unitarily equivalent to a free field theory.*

It will be shown in the following that the requirement of temperateness of generators (Schwartz distributions, equivalent to the existence of a translation covariant domain [77]) is very strong, it only allows integrable models and integrability in QFT can only be realized in d=1+1. Note that Wightman fields are assumed to be operator-valued temperate distributions. Hence the theorem says that even in case of a weak localization requirement as wedge-localization, one cannot find interacting operators with reasonable domain properties. However any QFT permits wedge-localized *nontemperate* generators [77]. The theorem has a rich pre-history which dates back to Furry and Oppenheimer's observation (shortly after Heisenberg's discovery of localization-cause vacuum polarization) that Lagrangian interactions always lead to fields which, if applied to the vacuum, inevitably create an infinite particle-antiparticle polarization cloud in addition to the desired one-particle state.

The only remaining possibility to maintain a relation between a **p**olarization-**f**ree **g**enerator (PFG) leading to a pure one-particle state and a localized operator (representing the field side) has to go through the bottleneck of nontemperate PFG generators of wedge-localized algebras; this is all which remains of the functorial particle-field relation in the absence of interactions.

For the on-shell construction one needs also a relation between multiparticle states and (naturally nontemperate) operators from wedge algebra. The idea is to construct a kind of *"emulation" of free incoming fields* (~multi-particles states) restricted to a wedge regions *inside the interacting wedge algebra* as a replacement for the nonexisting second quantization functor. As the construction of one-particle PFGs this is achieved with the help of modular localization theory.

The starting point is a *bijection* between wedge-localized incoming fields operators and interacting operators. This bijection is based on the equality of the dense subspace which these operators from the two different algebras create from the vacuum. Since the domain of the Tomita $S$ operators for two algebras which share the same modular unitary $\Delta^{it}$ is the same, a vector $\eta \in domS \equiv domS_{\mathcal{A}(W)} = dom\Delta^{\frac{1}{2}}$ is also in $domS_{\mathcal{A}_{in}(W)} = \Delta^{\frac{1}{2}}$ (in [77] it was used for one-particle states). In more explicit notation, which emphasizes the bijective nature, one has

$$A\left|0\right\rangle = A_{\mathcal{A}(W)}\left|0\right\rangle,\ A \in \mathcal{A}_{in}(W),\ A_{\mathcal{A}(W)} \in \mathcal{A}(W) \tag{32}$$

$$S(A)_{\mathcal{A}(W)}\left|0\right\rangle = (A_{\mathcal{A}(W)})^{*}\left|0\right\rangle = S_{scat}A^{*}S_{scat}^{-1}\left|0\right\rangle,\ S = S_{scat}S_{in}$$

$$S_{scat}A^{*}S_{scat}^{-1} \in \mathcal{A}_{out}(W) \tag{33}$$

Here $A$ is either an operator from the wedge localized free field operator algebra $\mathcal{A}_{in}(W)$ or an (unbounded) operator affiliated with this algebra (e.g. products of incoming free fields $A(f)$ smeared with $f,\ suppf \in W$); $S$ denotes the Tomita operator of the interacting algebra $\mathcal{A}(W)$. Under the assumption that the dense set generated by the dual wedge algebra $\mathcal{A}(W)'\left|0\right\rangle$ is in the domain of definition of the bijective defined "emulats" (of the wedge-localized free field operators inside its interacting counterpart), the $A_{\mathcal{A}(W)}$ are uniquely defined; in order to be able to use them for the reconstruction of $\mathcal{A}(W)$ the domain should be a *core for the emulats.* Unlike smeared Wightman fields, the emulats $A_{\mathcal{A}(W)}$ do not define a polynomial algebra, since their unique existence does not allow to impose additional properties; in fact they only form a vector space and the associated algebras have to be constructed by spectral theory or other means to extract an algebra from a vector space of closed operators.

Having settled the problem of uniqueness, the remaining task is to determine their action on wedge-localized multi-particle vectors and to obtain explicit formulas for their particle formfactors. All these problems have been solved in case the domains of emulats are invariance under translations; in that case the emulats possess a Fourier transform [77]. This requirement is extremely restrictive and is only compatible with d=1+1 elastic two-particle scattering matrices of integrable models[46]; in fact it should be considered as the *foundational definition of integrability of QFT in terms of properties of wedge-localized generator* [10].

Since the action of emulats on particle states is quite complicated, we will return to this problem after explaining some more notation which is useful for formulating the crossing identity in connection with its KMS counterpart and to remind the reader of how these properties have been derived in the integrable case.

For integrable models the wedge duality requirement [10] leads to a unique solution (the Zamolodchikov-Faddeev algebra), whereas for the general non-integrable case we will present arguments, which together with the comparison with integrable case determine the action of emulates on particle states. The main additional assumption is that the only way in which the interaction enters the this construction of bijections is through the S-matrix[47]. With this assumption the form of the action of the operators $A_{\mathcal{A}(W)}$ on multiparticle states is fixed. The ultimate check of its correctness through the verification of wedge duality ([10]) is left to future investigations.

Whereas domains of emulats in the integrable case are translation invariant [77], the only domain property which is *always* preserved in the general case is the invariance of the domain under the subgroup of those Poincaré transformations which leave W invariant. In contrast to QM, for which integrability occurs in any dimension, integrability in QFT is restricted to d=1+1 factorizing models [10].

A basic fact in the derivation of the crossing identity, including its analytic properties which are necessary in order to return to the physical boundary, is the *cyclic KMS property.* For three operators affiliated with the interacting algebra $\mathcal{A}(W)$, two of them being emulates of incoming operators[48] it reads:

---

[46]This statement, which I owe to Michael Karowski, is slightly stronger than that in [77] in that that higher elastic amplitudes are combinatorial products of two-particle scattering functions, i.e. the only solutions are the factorizing models.

[47]A very reasonable assumption indeed because this is the only interaction-dependent object which enters as a relative modular invariant the modular theory for wedge localization.

[48]There exists also a "free" KMS identity in which $B$ is replaced by $(B)_{\mathcal{A}_{in}(W)}$ so everything refers to the algebra $\mathcal{A}_{in}(W)$. The derivation of the corresponding crossing identity is rather simple [10] and its use is limited to problems of writing iterating fields as a series of Wick-ordered product of free fields.

$$\left\langle 0|BA^{(1)}_{\mathcal{A}(W)}A^{(2)}_{\mathcal{A}(W)}|0\right\rangle \overset{KMS(\mathcal{A}(W))}{=} \left\langle 0|A^{(2)}_{\mathcal{A}(W)}\Delta BA^{(1)}_{\mathcal{A}(W)}|0\right\rangle \tag{34}$$
$$A^{(1)} \equiv: A(f_1)...A(f_k) :,\ A^{(2)}_{in} \equiv: A(f_{k+1})...A(f_n) :,\ suppf_i \in W$$

where in the second line the operators were specialized to Wick-ordered products of smeared free fields $A(f)$ which are then emulated within $\mathcal{A}(W)$. Their use is necessary in order to convert the KMS relation for $\mathcal{A}(W)$ into an identity of *particle formfactors* of the operator $B \in \mathcal{A}(W)$. If the bijective image acts on the vacuum, the subscript $\mathcal{A}(W)$ for the emulats can be omitted and the resulting Wick-ordered product of free fields acting on the vacuum describe a multi-particle state in $\hat{f}_i$ momentum space wave functions. The roof on top of $f$ denotes the wave function which results from the forward mass shell restriction of the Fourier transform of W-supported test function. The result are wave functions in a Hilbert space of the graph norm $\left(\hat{f}, (\Delta+1)\,\hat{f}\right)$ which forces them to be analytic in the strip $0 < Im\theta < \pi$.

The easy part in the particle transcription of the KMS relation (34) is the right hand side. Letting the hermitian conjugate of $\Delta^{\frac{1}{2}}A^{(2)}_{\mathcal{A}(W)}$ act on the bra vacuum and using its modular representation (34) one obtains an outgoing n-k state in which the particles have been changed into their antiparticles; the application of the remaining $\Delta^{\frac{1}{2}}$ amounts to an analytic continuation of the antiparticle rapidities by $i\pi$ so that the net result is the analytically continued formfactor of $B$ between a n-k outgoing bra antiparticle state and an incoming k-particle state.

As will be seen the left hand side in (34) can, under special ordering conditions for the n rapidities, be replaced by an n-particle incoming vector which then represents the desired crossing relation. For simplicity of notation we specialize to d=1+1 in which case neither the wedge nor the mass-shell momenta have a transverse component and particles are characterized by their rapidity. Up to now the KMS relation only reads

$$\int ..\int \hat{f}_1(\theta_1)...\hat{f}_1(\theta_n)F^{(k)}(\theta_1,...,\theta_n)d\theta_1...d\theta_n = 0 \tag{35}$$
$$F^{(k)}(\theta_1,..,\theta_n) := \left\langle 0\left|BA^{(1)}_{\mathcal{A}(W)}(\theta_1,..,\theta_k)\right|\theta_{k+1},..,\theta_n\right\rangle_{in} -$$
$$-\ {}_{out}\left\langle\bar{\theta}_{k+1},..,\bar{\theta}_n\left|\Delta^{\frac{1}{2}}B\right|\theta_1,..,\theta_k\right\rangle_{in}$$

where $\bar{\theta}$ refers to antiparticle rapidities and the $\Delta^{\frac{1}{2}}$ of $\Delta$ was used to re-convert the antiparticle wave functions in the outgoing bra vector back into the original particle wave functions [10].

There are two steps which remain to be shown

1. For ordered rapidities $\theta_1 > ... > \theta_n$

$$\left\langle 0\left|BA^{(1)}_{\mathcal{A}(W)}(\theta_1,..,\theta_k)\right|\theta_{k+1},..,\theta_n\right\rangle_{in} = \left\langle 0\left|B\right|\theta_1,..,\theta_n\right\rangle_{in}$$

2. $F^{(k)}$ is locally square integrable

The first property is part of an analytic interpretation: the n-particle component of a local operator is the boundary value of a multivalued function in the multivariable $\theta$ space. One uses the statistics degeneracy of the n-particle vector to encode it into the $\theta$-ordering; any other order correspond to another boundary value of the formfactor which results from the particular analytic continuation used to arrive at the re-ordered $\theta$-configuration. Its physical interpretation is very different from the original n-particle interpretation in fact in general the new object represents a new state[49]. The derivation of the crossing identity does not require an operational identification of other boundary values because the ordering of the $\theta$ remains fixed (fixed $\mathrm{L}^2$ wave functions with ordered support) in the derivation of the crossing identity. The only place where a physical idea enters in addition to the KMS identity is in the assumption that the singularies near the boundary are exhaused by the known multiparticle threshold cuts. Without knowing anything about the distributional nature of boundary values (in this case the local $L^2$ integrability) one cannot use the $L^2$ denseness property of wedge-localized wave functions.

For the formulation of an on-shell construction project one needs more. The only known way goes via an assumption about an operational interpretation of the analytic reordering i.e. about the operational meaning of analytic $\theta$-reorderings in states and their possible dependence on the analytic path taken to get to the reordered configuration. This is tantamount to knowing the action of a PFG or a more general emulat on particle states beyond the vacuum. The guiding idea is that if one rapidity, say the first one in an n-particle state, is outside its ordered position then the commutaion with a k-particle cluster which is necessary to get it their only depends on this k-cluster and is described in terms of a "grazing shot" S-matrix in which there is no direct interaction within the cluster but only that part of the interaction which the $\theta_1$ causes in order to bring it into its ordered position. The implementation of this idea requires some new concepts and necessitates abbreviated notation in order to avoid messy formulas. It did not yet path its crucial test of "wedge duality" which would show its correctness.

However for d=1+1 integrable models it undergoes a significant simplification which allows to check the wedge-duality property. In this case the on-shell generating PFGs of the interacting wedge algebra fulfill the commutation relations of the Zamolodchikov-Faddeev algebra [68][69] and can be used to construct the compact localized double cone algebra and in this way show the mathematical existence of QFTs with realistic strictly renormalizable short-distance behavior [23] (the first time in the long almost 90 year old history of QFT).

# 5 Resumé and concluding remarks

This paper consists of a critical part which analyses the conceptual situation of more than 4 decades of ST, and a part which proposes a radically new view about interacting higher spin particles and gauge theories in particular. Here they appear together because both are new insights which follow from the principle of modular localization which stands for a recent conceptually intrinsic and mathematically concise formulation of causal localization

[49] For explanatory simplicits we use the terminology "state", but in reality we talk about what happens to formfactors when their particle rapidities are being analytically continued.

in QT.

The important project of a mass-shell based top-to-bottom approach in particle theory took a wrong turn when, as a consequence of the insufficient understanding of the relation between on-shell analytic properties and the intrinsic localization properties of local quantum physics at that time, the dual model crossing was mistakenly accepted as describing the on-shell particle crossing. In order to underline the subtlety of this issue it was useful to go back to the beginnings of QFT and point to a non-understood aspect of the E-J conundrum: the modular localization property as the defining property of LQP i.e. of QFT unchained from quantization.

Looking back at the attempts at S-matrix based on-shell construction attempts of the 60s with present hindsight, one realizes that there was not much of a chance at that time for understanding the subtle role of the particle crossing property in such a project. As argued in section 2, there had been however a missed chance to notice that the crossing of *the meromorphic dual model functions has nothing to do with the on-shell dynamics of the S-matrix and formfactors*. After all the infinite component positive energy one-particle representation of the Poincaré group (the superstring representation), which was constructed on the irreducible oscillator algebra of a ten-component supersymmetric chiral current model, was nothing else than a solution of the "infinite component field equation" project of the 60s. This project, which really dates back to Majorana, was pursued by several people at the same time as the dual model, but the "dynamic infinite component field" community remained unaware about the string theorists solution of their problem; presumably because they mistakenly believed that the objects behind the dual model and ST were genuine spacetime strings whereas they were looking for infinite component pointlike fields. History will however be less lenient with physicists who used their status and charisma to uncritically market the new product in terms of slogans ("string theory is a piece of 21st century physics that fell by chance into the 20th century" and "string theory is the only game in town"). They bear some responsibility for the past and present situation which led to deep schism in particle theory. The mysterious *picture puzzle* aspect of the relation between the $(d, s)$ scale-dimension spectrum in a conformal QFT and $(m^2, s)$ Poincaré group representation spectrum, which created the impression of having come across a deep new relation, accounts for mitigating circumstances.

What made the situation even more muddled is the fact that mathematicians were able to abstract from the rather loose pictures of the string theorists valuable mathematical ideas, which in many cases the string theorists in turn took as a confirmation that they were working on a deep, albeit somewhat mysterious new theory (the longed for "theory of everything").

The *correct* understanding of particle crossing was not possible without perceiving the new role of the S-matrix as *a relative modular invariant* between the free incoming and the interacting wedge-localized subalgebra. In this way the derivation of the particle crossing identity became an important part of a new constructive top-to-bottom approach which starts from the classification and construction of generators of wedge-localized algebras from a known S-matrix[50] and ends with the net of compact localized from the existence

[50] Apart from the bootstrap construction of scattering functions for integrable models, the construction of an S-matrix cannot be separated from the construction of the wedge generator using the sytem of equations which follow from wedge duality. The hope is that the combined on-shell equations, unlike

of nontrivial intersections and their possible generating quantum fields. It had its biggest success for integrable models, for which it leads to existence proofs and provides the setting for ongoing explicit calculations. Integrable models are limited to d=1+1 dimensions but they present an interesting "theoretical laboratory" in order to test ideas about general models of QFT.

A comprehensive analysis of the causes of the existing deep schism within particle theory is not possible without looking also at sociological aspects. The appearance of wrong or useless theoretical constructs in a highly speculative area as particle theory is nothing new; the real problem is to understand why the dual model and ST, unlike numerous other failed ideas ("peratization", "Reggeization", SU(6), infinite component fields, Lee-Wick theory,...), holds on for almost 5 decades despite demonstrable misunderstandings and errors. A possible answer is that the foundational knowledge about its conceptual roots as an *on-shell construction project in local quantum physics* got lost after almost 5 decades. The awareness about the subtlety of analytic on-shell properties was still present in the early days of the S-matrix project (in the aftermath of the sucessful dispersion relation project) but were lost in the "picture puzzle" aspect of the dual model.

When, as a result of new ideas about analytic on-shell properties and their algebraic formulation from modular localization, the on-shell construction project returned at the end of the 90s, string theory had already lost its connection to its own roots. Already during the 80s ST begun to enter conferences and journals under its own label which disconnected it from its roots from in the strong interaction S-matrix project of the 60s. In this deliberate ahistorical self-presentation, it succeeded to convince many newcomers to particle theory that it presents the wave of the future (theory of everything), with QFT being assigned the role of a footnote. The conceptual differences between new foundational insights about on-shell constructions as presented in the present paper and ST became irreconcilable.

Among the theoreticians who followed the foundational developments of local quantum physics it is hard to meet anybody who does not know that ST and most of its derivatives (the Maldacena conjecture, brane physics, embeddings of one QFT into a higher one and its inverse: dimensional Kaluza-Klein reduction) are results of a conceptual flaw which resulted from a muddled view about localization in QFT versus that in QM. Attempts to explain to string theorists why the Maldacena conjecture is incorrect (The AdS-correspondence is incapable to relate two *physically acceptable* models) end always in impasse; either because the concepts used are outside the conceptual understanding of ST, or the discussion ends by claiming that the "German correspondence" (referring to Rehren's theorem) has no bearing on Maldacena's conjecture. For the first time in the history of particle physics a whole community got into a standoff situation in which its own conceptual resources are insuffient to get out of self-created scientific isolation.

Perhaps the schism has even deeper philosophical roots in the way particle research was conducted. Since Dirac's successful extraction of antiparticles from the later abandoned "hole theory" (contains no processes involving vacuum polarization), the method of research consisted in starting a computation and thinking about necessary modification "as one moves along". Often correct discoveries were made in settings which later turned

the standard off-shell perturbation theory, permit a convergent iteration which determines the S-matrix together with the wedge generators.

out to be incorrect. This trial and error method was for several decades extremely successful; most of the impressive results in particle physics after world war II were obtained in this way. More foundational directed research projects also existed parallel to this mainstream method; the oldest project was Wigner's 1939 classification of one-particle wave function spaces in terms of the representation theory of the Poincaré group, followed by Wightman's operator-valued distribution setting (shortly after Laurant Schwartz pathbreaking mathematical work on singular functions) and by Haag's 1957 formulation of "local quantum physics" in terms of nets of localized operator algebras. But there was little mainstream motivation for getting interested in such problems as long as the "compute, think and correct" way of conducting research was successful[51].

Cul de sac situations occasionally caused by ideas with little or no foundational support were usually cleared up within the well-functioning traditional European "Streitkultur" (represented by great figures as Pauli, Jost, Lehmann, Kallen, Landau,..) which at that time also took roots in the US (Oppenheimer, Feynman, Schwinger, Dyson,..). But this way of keeping viable progress going disappeared in the 70s. It may not be accidental, that after developing the Standard Model within the setting of gauge theories, the rate of genuine progress slowed down despite an increase in publications. In fact most of the problems one confronts nowadays (the Higgs issue, long distance behavior, the precise meaning of asymptotic freedom,...) were formulated and discussed in the 70s. This suggests that the mentioned method of conducting research in particle theory may have been exhausted, and that time has come for a new conceptual push. Fortunately at this time one is not empty-handed, LQP has matured and is now ready to make contact with important unsolved problems of interactions involving higher spins; the ideas in section 3 illustrate this point.

Nowhere has this dispute about the future of particle theory taken such extreme ideological forms as that about Maldacena's conjecture concerning the physical content of the AdS-CFT correspondence. As explained in section 3.4, the correct mathematical statement is that there is indeed an algebraic isomorphism, but that its physical content is severely limited by the fact that (depending on what side one starts), either the resulting CFT violates the causal completion property (leading to the from nowhere into the causal shadow entering of "poltergeist" degrees of freedom, see section 2), or the degrees of freedom of the resulting AdS theory remain below the cardinality of phase space degrees of freedom which is necessary to obtain nontrivial compactly localized subalgebras ("anemia" of degrees of freedom to populate a larger spacetime region). This is of course in agreement with the impossibility to illustrate the phenomenon in terms of Lagrangian models, since Lagrangian quantization is formally in agreement with both aspects of causality. Although Lagrangian quantization cannot reveal the mathematical existence since the renormalized series diverges, one believes that its structural properties correctly mirror foundational properties of QFT.

The insistence in the correctness of the Maldacena conjecture and the public use of derogatory terminology as "the German correspondence" for the proven theorem marks the sociological depth of the schism. For most particle physicists with an awareness about the past of their subject it is of course somewhat sad to see that the insights gained

[51] In more recent times Tegmark [99] proposed a more radical version.

in pre-electronic times into the connection between the causal completion property and the cardinality of phase space degrees of freedom (see section 2) have succumbed to the maelstrom of time in regard to the string-inspired generation. These insights had been obtained at a time when progress was still available without foundational knowledge about QFT. But now, when these post-quantization results are really needed (section 2) they are not available to the protagonists of the above conjecture and related subjects.

The situation is not so dissimilar from that in the financial markets; at the time when the tools of deregulated capitalism were working, hardly anybody was interested to listen to alternatives for what to do when one day they start tearing society apart. Apparently not even surreal consequences [83] are able to prevent people from being addicted to wrong conjectures as long as there is a sufficiently large community of subscribers; the only thing which would be the beginning of the end of the ST (and its derivatives) community is if one of its main supporters and updaters begins to have scrupels; but after more than 3 decades of investments in ST this is even less probable than a banker developing doubts about the ethical aspects of financial capitalism.

There is hardly anything more bizarre than the idea that we are living in a dimensionally reduced 10 dimensional target space of a chiral conformal QFT. Attributing to this observation the role of a key for understanding of the universe is not much different than the ontological role *attributed to the number 42 as an answer to the ultimate question about "Life, the Universe, and Everything"* in Douglas Adam's well-known scientific fiction comedy "the hitchhiker's guide through the galaxy".

In a way it is very fitting that a prize, which has been donated by somebody [84] who profited from this kind of capitalism, is given to the kind of unproductive but entertaining ST influenced particle theory which is sustained by ignorance about prior foundational results. It opens the possibility to physicists to get rich in the same way as the financial players to which the sponsor of this prize belongs, namely by creating unproductive toxic, and in case of particle physics, bizarre inventions. Usually the critique of attribution of highly lucrative prizes to less than Nobel worthy observations can be dismissed as resulting from envy. But in the mentioned cases, the fact that some of these observations are in variance with known facts raises series doubts about whether modern physics can maintain its status it had since Einstein, Heisenberg and others in the new social surrounding of unchained capitalism.

Never in the history of physics before has an area of research lend itself that easily to be used in entertainment and cinema as ST and its bizarre but entertaining off-spring as extra dimensions and dimensional reduction [87]; the ST saga has been spread worldwide on television by the former string-theorist Brian Green [88].

To feel the depth of the crisis into which large parts of particle theory has fallen, it is helpful to be reminded of a quotation from Einstein's talk in the honor of Planck [89].

*In the temple of science are many mansions, and various indeed are they who dwell therein and the motives that have led them thither. Many take to science out of a joyful sense of superior intellectual power; science is their own special sport to which they look for vivid experience and the satisfaction of ambition; many others are found in the temple who have offered the product of their brains on this altar for purely utilitarian purposes. Were an angel of the Lord to come and drive all these people belonging to these two categories out of the temple, the assemblage would be seriously depleted, but there would* *still be some men, of present and past times, left inside. Our Planck is one of them, and that is why we love him. ...*

But where has Einstein's *Angel of the Lord*, the protector of the temple of science, gone in the times of string theory and all its derivatives? With the continuation of the old Streitkultur we would have had a chance to get out of this, in fact we may not even have gotten into it.

Given that sociological situation with respect to ST and its derivatives, one should not expect changes in the foreseeable future. It is more probably that the ongoing progress about renormalization theory from string-localized higher spin fields, in particular new insights about renormalization of vectorpotentials in massive and massless s=1 models (indicated in section 3), could achieve such a revolution. The critique of ST, a new on-shell project and a first account of this ongoing revolution in renormalization theory have their conceptual roots in modular localization. But it seems that only the last topic has the power to end the present schism in particle theory.

In the past it was easy to ignore the existing critical remarks, since no concerted efford at a scientific critique of ST existed; people just expressed opinions about its bizarre consequences or pointed to the many decades which passed without any experimentally accessible consequences (Woit) or on other sociological-philosophical points (Smolin). Actually in an older paper Smolin together with Arnsdorf came quite close to raise an important scientific point [90]. These authors, standing on the shoulders of Rehren, pointed at a kind of conundrum between the consequences of the string-induced Maldacena conjecture [83] and Rehren's theorem [52]. This is precisely connected to the degrees of freedom problem explained in section 2 and 3.

One can ask the question whether it is possible to slightly modify the AdS-CFT setting, so that an appropriately reformulated Maldacena's conjecture can be saved from the enormous pile of publications by establishing harmony with the rigorous theorem. This is precisely the question Kay and Ortiz asked [91]. Taking their cue from prior work on the correspondence principle of Mukohyama-Israel as well from 't Hoofts brick-wall idea[52] [92], these authors start with a Hartle-Hawking-Israel like pure state on an imagined combined matter + gravity dynamic system. They then propose to equate the AdS side of a hypothetical conformal invariant supersymmetric Yang-Mills model with the restriction of the H-H-I state to a matter subsystem which is in accordance with Rehren's theorem. That this can be achieved is not very plausible (as the authors themselves admit).

Concerning defences of ST, one may refer to a recent paper by Duff [93] "String and M-Theory: Answering the Critics" within a project "Forty Years Of String Theory" where the author basically musters all the names of well-known people (besides the hard core string theorists) who, guided by their natural intellectual curiosity looked at ST and whose first (and in most cases only) reaction was quite positive. Feynman's name does not appear there, which may be related to his well-known accusation of string theorist to counter scientific critique by inventing excuses.

Further critical remarks will be left to the philosophers and historians of physics; the 50

[52]This idea seems to imply a conjecture about the dependence of localization entropy of a fuzzy surface which is expected to result as a theorem from the degrees of freedom picture which leads to the split property.

years of unopposed derailment of parts of particle theory will provide ample material to be analyzed. The future capacity of QFT stands in contrast to the present sociological-caused schism within particle theory. Hopefully the present work succeeds to draw attention to the enormous potential which the good old QFT still has in store for us if we are willing to engage in a pursuit of its foundations.

*Acknowledgement*: I am indebted to Bernard Kay for pointing to his attempt to solve the Arnsdorf-Smolin conundrum. Thanks go also to Raymond Stora for explaining the remaining problems of perturbative renormalization theory and to Jens Mund for innumerous discussions about string-localized fields.

# References

[1] A. Duncan and M. Janssen, *Pascual Jordan's resolution of the conundrum of the wave-particle duality of light*, arXiv:0709.3812

[2] B. Schroer, *The Einstein-Jordan conundrum and its relation to ongoing foundational research in local quantum physics*, to be published in EPJH, arXiv:1101.0569

[3] R. Haag, *Local Quantum Physics*, Springer 1996

[4] W. G. Unruh, *Notes on black hole evaporation*, Phys. Rev. **D14**, (1976) 870

[5] G. Sewell, Ann. Phys. **141**, (1982) 201

[6] D. Buchholz and C. Solveen, *Unruh Effect and the Concept of Temperature*, arXiv:1212.2409

[7] B. Schroer and H.-W. Wiesbrock, Rev. Math. Phys.**12** (2000) 461, arXiv:hep-th/9901031

[8] R. Kaehler and H.-P. Wiesbrock, *Modular theory and the reconstruction of four-dimensional quantum field theories*, Journal of Mathematical Physics **42**, (2001) 74

[9] B. Schroer, *Bondi-Metzner-Sachs symmetry, holography on null-surfaces and area proportionality of "light-slice" entropy*, Foundations of Physics **41**, 2 (2011), 204, arXiv:0905.4435

[10] B. Schroer, *The foundational origin of integrability in quantum field theory*, Foundations of Physics **42**, (2012) 1481, arXiv:1109.1212

[11] B. Schroer, *Studies in History and Philosophy of Modern Physics* **41** (2010) 104–127, arXiv:0912.2874

[12] H. Epstein and V. Glaser, Ann. Inst. Henri Poincaré A XIX, (1973) 211

[13] W. Heisenberg, *Die beobachtbaren Grössen in der Theorie der Elementarteilchenphysik*, Z. Physik **120**, (1943) 513

[14] J. Bros, H. Epstein and V. Glaser, Com. Math. Phys. **1**, (1965) 240

[15] S. Mandelstam, Phys. Rev. **112**, (1958) 1344 and Phys. Rev. **115**, (1959) 1741

[16] G. Veneziano, Nuovo Cim. A **57**, (1968) 190

[17] G. Mack, *D-dimensional Conformal Field Theories with anomalous dimensions as Dual Resonance Models*, arXiv:0907.2407

[18] G. Mack, *D-independent representation of Conformal Field Theories in D dimensions via transformation to auxiliary Dual Resonance Models. Scalar amplitudes*, arXiv:0907.2407v1

[19] R. Brunetti, D. Guido and R. Longo, *Modular localization and Wigner particles*, Rev. Math. Phys. **14**, (2002) 759

[20] J. Mund, B. Schroer and J. Yngvason, *String-localized quantum fields and modular localization*, CMP **268** (2006) 621, math-ph/0511042

[21] H-J. Borchers, *On revolutionizing quantum field theory with Tomita's modular theory*, J. Math. Phys. **41**, (2000) 8604

[22] B. Schroer, Annals Phys. **275** (1999) 190

[23] G. Lechner, *An Existence Proof for Interacting Quantum Field Theories with a Factorizing S-Matrix*, Commun. Mat. Phys. **227**, (2008) 821, arXiv.org/abs/math-ph/0601022

[24] R. Haag, *Discussion of the 'axioms' and the asymptotic properties of a local field theory with composite particles* (historical document), Eur. Phys. J. H 35, 243–253 (2010)

[25] K. Fredenhagen, *Lille 1957: The birth of the concept of local algebras of observables*, Eur. Phys. J. H 35, 239–241 (2010)

[26] B. Schroer, *Pascual Jordan's legacy and the ongoing research in quantum field theory*, Eur. Phys. J. H **35**, (2011) 377, arXiv:1010.4431

[27] R. F. Streater and A. S. Wightman, PCT Spin&Statistics and all that, New York, Benjamin 1964

[28] R. Haag and B. Schroer, *Postulates of Quantum Field Theory*, J. Mat. Phys. **3**, (1962) 248

[29] D. Buchholz and E. Wichmann, Commun. Math. Phys. **106** (1986) 321

[30] D. Buchholz, C. D'Antoni and R. Longo, *Nuclear maps and modular structures. I. General properties*, J. Funct. Anal. **88** (1990) 233

[31] G. C. Hegerfeldt, Causality problems in Fermi's two atom system, Phys. Rev. Lett. **72**, (1994) 596-599

[32] D. Buchholz and J. Yngvason, *There are no causality problems for Fermi's two atom system*, Physical Review Letters **73**, (1994) 613-616

[33] B. Schroer, *An alternative to the gauge theory setting*, Found. of Phys. **41**, (2011) 1543, arXiv:1012.0013

[34] B. Schroer, *Unexplored regions in QFT and the conceptual foundations of the Standard Model*, arXiv:1010.4431

[35] S. Hollands and R. M. Wald, General Relativity and Gravitation **36**, (2004) 2595

[36] J Polchinski, *String theory I*, Cambridge University Press 1998

[37] D. Buchholz, *New Light on Infrared Problems: Sectors, Statistics, Spectrum and All That*, arXiv:1301.2516

[38] E. Majorana, Teoria relativistica di particelle con momentum internisico arbitrario, Nuovo Cimento 9, (1932) 335

[39] N. N. Bogoliubov, A. Logunov, A. I. Oksak and I. T. Todorov, *General principles of quantum field theory*, Dordrecht Kluwer

[40] D. A. Lowe, Phys. Lett. B 326, (1994) 223

[41] E. Martinec, Class. Quant. Grav. **10**, (1993) 187

[42] R. Jost: TCP-Invarianz der Streumatrix und interpolierende Felder, Helvetica Phys. Acta **36**, (1963) 77

[43] A. A. Belavin, A. M. Polyakov and A. B. Zamolodchikov, Nucl. Phys. B **241** (2), (1984) 333

[44] M. Lüscher and G. Mack, Comm. Math. Phys. **41**, (1975) 203

[45] B. Schroer and J. A. Swieca, Phys. Rev. D **10**, (1974) 480

[46] P. Di Vecchia, *The birth of string theory*, Lect. Notes Phys.**737**, (2008) 59-118, arXiv 0704.0101

[47] B. Schroer, *Causality and dispersion relations and the role of the S-matrix in the ongoing research*, Foundations of Physics **42**, (2012) 1481, arXiv:1107.1374

[48] D. Buchholz, G. Mack and I. Todorov, Nucl.Phys. B, Proc. Suppl. **5B**, (1988) 20

[49] C.P. Staskiewicz, Die lokale Struktur abelscher Stromalgebren auf dem Kreis, Thesis at Freie Universitaet Berlin (1995)

[50] Y. Kawahigashi and R. Longo, Adv. Math. **206** (2006) 729, arXiv:math/0407263

[51] D. Buchholz and K. Fredenhagen, Commun. Mat. Phys. **84**, (1982) 1

[52] K-H. Rehren, *Algebraic Holography*, Annales Henri Poincaré**1**, (2000) 607, arXiv:hep-th/9905179

[53] D. Bahns, K. Rejzner and J. Zahn, *The effective theory of strings*, arXiv:1204.6263

[54] K. Fredenhagen and K. Rejzner. *Local covariance and background independence*, arXiv:1102.2376

[55] D. Bahns, J. Math. Phys. **45**, (2004) 4640

[56] S. Weinberg, *The Quantum Theory of Fields I,* Cambridge University Press

[57] J. Yngvason, Zero-mass infinite spin representations of the Poincar´e group and quantum field theory, Commun. Math. Phys. **18** (1970), 195

[58] M. Plaschke and J. Yngvason, Journal of Math. Phys. **53**, (2012) 042301

[59] S. Mandelstam, Phys. Rev. **175**, (1968) 1518

[60] K. Bardackci and B. Schroer, *Local Approximations in Renormalizable and Nonrenormalizable Theories II*, J. Math. Phys **7**, (1966) 16.

[61] G. Björck, Arkiv för Mathematik **6**, (1965) 351

[62] A. Jaffe, Phys. Rev. **158**, (1967) 1454

[63] J. Mund, String-localized massive vector Bosons without ghosts: the example of massive QED, to appear

[64] B. Schroer, Quadratic dependence on string-localized massive vectormesons: the example of scalar QED, in preparation

[65] G. Scharf, Quantum Gauge Theories, a true ghost story, John Wiley&Sons, INC 2001

[66] J. H. Lowenstein and B. Schroer, Phys. Rev. **D7**, (1975) 1929

[67] L. Fassarella and B. Schroer, *Wigner particle theory and local quantum physics*, J. Phys. A **35**, (2002) 9123-9164

[68] B. Schroer, *Modular localization and the bootstrap-formfactor program*, Nucl. Phys. **B499**, 1997, 547, hep-th/9702145.

[69] B. Schroer, *Modular localization and the d=1+1 formfactor program*, Annals of Physics **295**, (1999) 190

[70] R. Kaehler and H.-P. Wiesbrock, *Modular theory and the reconstruction of four-dimensional quantum field theories*, Journal of Mathematical Physics **42**, (2001) 74-86

[71] B. Schroer, *Particle physics in the 60s and 70s and the legacy of contributions by J. A. Swieca*, Eur.Phys.J.H **35**, (2010) 53, arXiv:0712.0371

[72] J. Schwinger, Trieste Lectures, 1962, p. 89, I.A.E.A., Vienna 1963

[73] A. J. Swieca, *Charge screening and mass spectrum*, Phys. Rev. D **13**, (1976) 312

[74] J. Sucher and C. H. Woo, Phys. Rev. D **8**, (1973) 2721

[75] M. Duetsch, J. M. Gracia-Bondia, F. Scheck, J. C. Varilly, *Quantum gauge models without classical Higgs mechanism*, arXiv:1001.0932

[76] J. Mund, *An Algebraic Jost-Schroer Theorem for Massive Theories*, arXiv:1012.1454

[77] H. J. Borchers, D. Buchholz and B. Schroer, Commun. Math. Phys. **219** (2001) 125

[78] H. Babujian, A.Fring, M. Karowski and A. Zapletal, Nucl. Phys. B **538**, (1990) 535

[79] Buchholz and S.J Summers, *Scattering in Relativistic Quantum Field Theory: Fundamental Concepts and Tools*, arXiv:math-ph/0405058

[80] D. Buchholz, Commun. math. Phys. **36**, (1974) 243

[81] D. Buchholz and J. Roberts, *New Light on Infrared Problems: Sectors, Statistics, Symmetries and Spectrum*, arXiv:1304.2794

[82] G. Lechner, *Deformations of quantum field theories and integrable models*, arXiv:1104.1948

[83] J. Maldacena, *The Illusion of Gravity*, Scientific American, (November 2005) page 56-63

[84] http://en.wikipedia.org/wiki/Fundamental_Physics_Prize

[85] J. Bros and J. Mund, *Braid group statistics implies scattering in three-dimensional local quantum physics*, arXiv:1112.5785

[86] http://golem.ph.utexas.edu/~distler/blog/archives/000987.html

[87] L. Randall on http://www.youtube.com/watch?v=Q5ikGoJTpMM

[88] http://www.ted.com/talks/brian_greene_on_string_theory.html

[89] http://www.scribd.com/doc/13093631/Einstein-in-His-Own-Words

[90] M. Arnsdorf and L. Smolin, *The Maldacena Conjecture and Rehren Duality*, arXiv:hep-th/0106073

[91] B. S. Kay and L. Ortiz, *Brick Walls and AdS/CFT*, arXiv:1111.6429

[92] G. 't Hooft, *On the quantum structure of a black hole*, Nucl. Phys. B **256**, (1985) 727

[93] M. J. Duff, *String and M-theory: answering the critics*, arXiv:1112.0788

[94] H. Babujian and M. Karowski, Int. J. Mod. Phys. **A1952**, (2004) 34, and references therein to the beginnings of the bootstrap-formfactor program

[95] J. Mund, Prog. Math. **251**, (2007) 199, arXiv:hep-th/0502014

[96] J. Mund, private communication

[97] R. M. Wald, *The Formulation of Quantum Field Theory in Curved Spacetime*, arXiv:0907.0416

[98] R. Haag and J. A. Swieca, *When does a quantum field theory describe particles?*, Math. Phys. **1**, (1965) 308

[99] M. Tegmark, *Shut up and calculate*, arXiv:0709.4024, and prior similar philosophical Zeitgeist contributions