Factorization Algebras vs AQFT

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Joint work with M. Benini and A. Schenkel [Commun. Math. Phys. (2019)]

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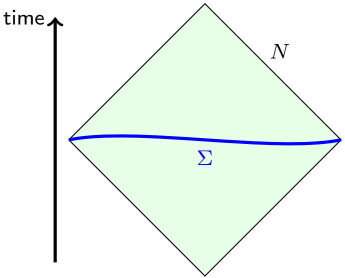
- **Objects** := oriented and time-oriented globally hyperbolic Lorentzian manifold N



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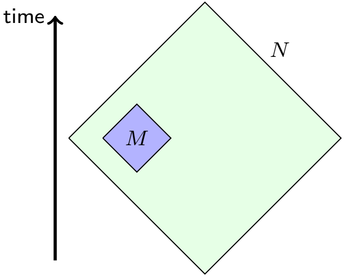

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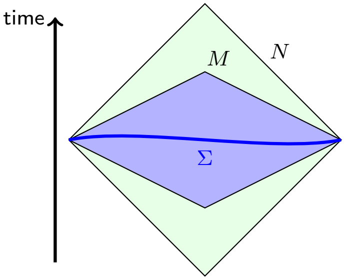

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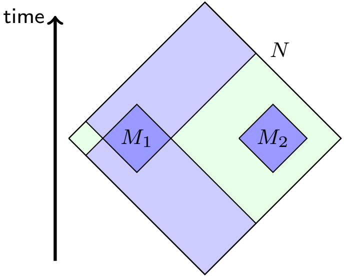

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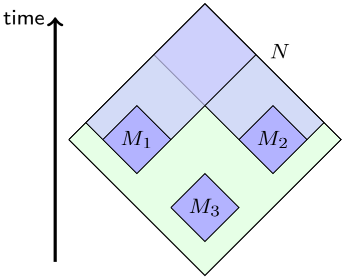

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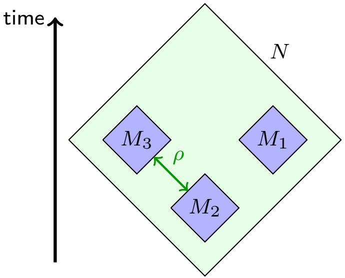

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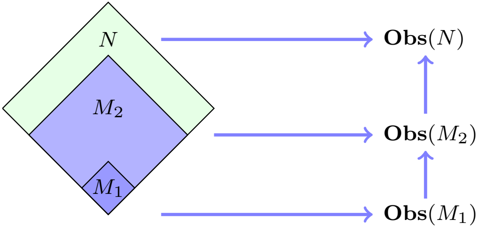
  (iv) **Time-orderable tuple:** $\underline{f} : \underline{M} \to N$ s.t. there exists $\rho \in \Sigma_n$ (time-ordering permutation) with $\underline{f}\rho = (f_{\rho(1)}, \dots, f_{\rho(n)}) : \underline{M}\rho \to N$ time-ordered

An intuitive idea: AQFT and tPFA

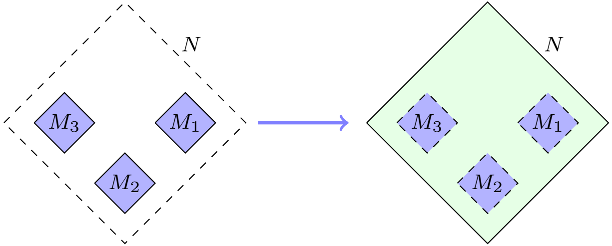
Roughly speaking a $\mathbf{QFT}$ on a Lorentzian manifold is an assignment of observables to open causally convex subsets in a functorial way.



In addition to assigning observables a $\mathbf{QFT}$ should come equipped with a rule on how to multiply certain observables. There exist different axiomatizations: $\mathbf{tPFA}$ and $\mathbf{AQFT}$.

An intuitive idea: AQFT and tPFA

- A tPFA assigns to every spacetime a **vector space** of observables and to every time-orderable tuple $\underline{f} : \underline{M} \to N$ a **factorization product** $\bigotimes_{i=1}^{n} \mathbf{Obs}(M_i) \to \mathbf{Obs}(N)$.



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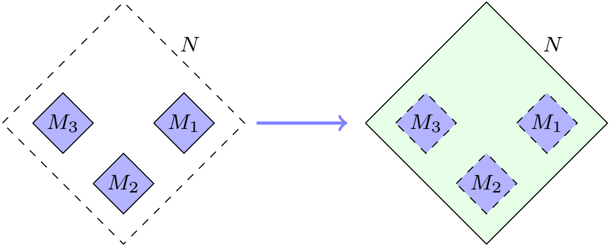

- An AQFT assigns to every spacetime M an object of $\mathbf{Alg}_{\mathsf{As}}(\mathbf{Vec}_{\mathbb{K}})$, i.e $\mathbf{Obs}(M)$ comes endowed with a product μ and a unit η.

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$$\begin{tikzcd}
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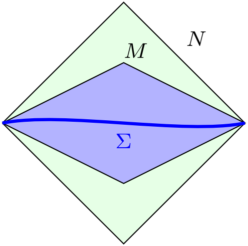
- **Morphisms:** A morphism $\zeta : \mathfrak{F} \to \mathfrak{G}$ is a family of linear maps $\zeta_M : \mathfrak{F}(M) \to \mathfrak{G}(M)$, for all $M \in \mathbf{Loc}$, that is compatible with the factorization products, i.e. $\zeta_N \circ \mathfrak{F}(\underline{f}) = \mathfrak{F}(\underline{f}) \circ \bigotimes_i \zeta_{M_i}$, for all $\underline{f} : \underline{M} \to N$.

The meaning of c and add

Def: $\mathfrak{F} \in \mathbf{tPFA}$ is called **Cauchy constant** if $\mathfrak{F}(f) : \mathfrak{F}(M) \xrightarrow{\cong} \mathfrak{F}(N)$ is isomorphism for all Cauchy morphisms $f : M \to N$.

- In particular the observables of N are fully determined by those of M. This condition should be thought as encoding a concept of **time evolution**.

  We denote by $\mathbf{tPFA}^{c}$ the full subcategory of $\mathbf{tPFA}$ formed by the Cauchy constant $\mathfrak{F} \in \mathbf{tPFA}$.



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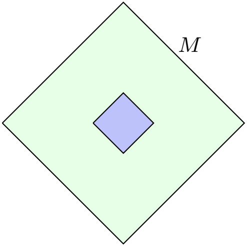

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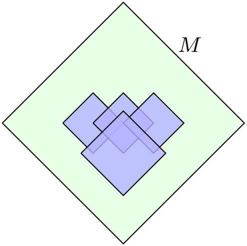

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$\mathfrak{F} \in \mathbf{tPFA}$ is called **additive** if

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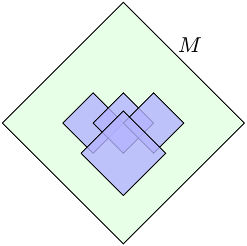

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Denote by $\mathbf{tPFA}^{\mathrm{add}} \subseteq \mathbf{tPFA}$ the full subcategory of additive tPFAs and by $\mathbf{tPFA}^{\mathrm{add,c}} \subseteq \mathbf{tPFA}$ the full subcategory of Cauchy constant additive tPFAs.

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- Additivity should be thought as encoding a concept of **compact support for observables**. This meaning that the obsevables on the whole region are determined by those on the relatively compact ones (this is to avoid issues at 'infinity').

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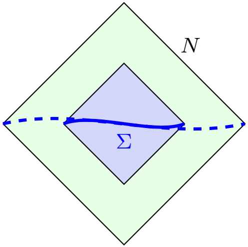

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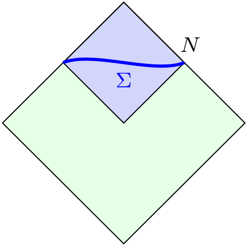
- This is not possible in general.



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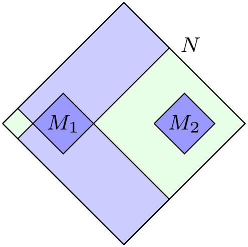
- The category $\mathbf{AQFT}$ is defined by the following data:
- **Objects**: Are the AQFTs on $\mathbf{Loc}$. An AQFT on $\mathbf{Loc}$ is a functor $\mathfrak{A} : \mathbf{Loc} \to \mathbf{Alg} := \mathbf{Alg}_{\mathsf{As}}(\mathbf{Vec})$ satisfying the **Einstein causality axiom**: For causally disjoint $(f_1 : M_1 \to N, f_2 : M_2 \to N)$,

$$\begin{tikzcd} \mathfrak{A}(M_1) \otimes \mathfrak{A}(M_2) \arrow[r, "\mathfrak{A}(f_1)\otimes\mathfrak{A}(f_2)"] \arrow[d, "\mathfrak{A}(f_1)\otimes\mathfrak{A}(f_2)"'] & \mathfrak{A}(N) \otimes \mathfrak{A}(N) \arrow[d, "\mu_N^{\mathrm{op}}"] \\ \mathfrak{A}(N) \otimes \mathfrak{A}(N) \arrow[r, "\mu_N"'] & \mathfrak{A}(N) \end{tikzcd}$$



- Any two spacelike separated observables commute with each other

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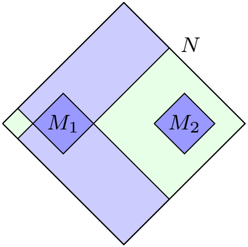
$$\begin{tikzcd} \mathfrak{A}(M_1) \otimes \mathfrak{A}(M_2) \arrow[r, "\mathfrak{A}(f_1)\otimes\mathfrak{A}(f_2)"] \arrow[d, "\mathfrak{A}(f_1)\otimes\mathfrak{A}(f_2)"'] & \mathfrak{A}(N) \otimes \mathfrak{A}(N) \arrow[d, "\mu_N^{\mathrm{op}}"] \\ \mathfrak{A}(N) \otimes \mathfrak{A}(N) \arrow[r, "\mu_N"'] & \mathfrak{A}(N) \end{tikzcd}$$



- Any two spacelike separated observables commute with each other
- **Morphisms**: given by natural transformations.
- Cauchy constancy and additivity can be also defined for AQFTs. Denote the corresponding full subcategories by $\mathbf{AQFT}^{\mathrm{c}}$, $\mathbf{AQFT}^{\mathrm{add}}$ and $\mathbf{AQFT}^{\mathrm{add,c}}$.

$\mathbf{AQFT}^{\mathrm{add},c} \to \mathbf{tPFA}^{\mathrm{add,c}}$

It is pretty straightforward and there is no need for Cauchy constancy and additivity at this level.

- Let $\mathfrak{A} \in \mathbf{AQFT}$. We want to define a corresponding $\mathfrak{F}_{\mathfrak{A}} \in \mathbf{tPFA}$.

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(2) For time-orderable $\underline{f} : \underline{M} \to N$ with time-ordering permutation $\rho \in \Sigma_n$, define the factorization product $\mathfrak{F}_{\mathfrak{A}}(\underline{f}) : \bigotimes_{i=1}^n \mathfrak{A}(M_i) \to \mathfrak{A}(N)$ by

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Prop: We obtain a functor $\mathfrak{F}_{(-)} : \mathbf{AQFT} \to \mathbf{tPFA}$ that restricts to Cauchy constant additive theories $\mathfrak{F}_{(-)} : \mathbf{AQFT}^{\text{add,c}} \to \mathbf{tPFA}^{\text{add,c}}$.

Comments on the Remark

Rem Suppose you define $\mathfrak{F}_{\mathfrak{A}}(\underline{f})$ as $\mu_N^{(n)} \circ \bigotimes_i \mathfrak{A}(f_i)$, in particular for $n = 2$ you obtain the following diagram

$$\begin{tikzcd} \bigotimes_{i=1}^{2} \mathfrak{A}(M_i) \arrow[r, "\mathfrak{F}_{\mathfrak{A}}(\underline{f})"] \arrow[d, "\text{do NOT permute}"'] & \mathfrak{A}(N) \\ \bigotimes_{i=1}^{2} \mathfrak{A}(M_i) \arrow[r, "\bigotimes_i \mathfrak{A}(f_i)"'] & \mathfrak{A}(N)^{\otimes 2} \arrow[u, "\mu_N^{(2)}"'] \end{tikzcd}$$

and suppose that M_1, M_2 are NOT causally disjoint. If $\mathfrak{F}_{\mathfrak{A}}$ was a $\mathbf{tPFA}$ then it would have to satisfy the equivariance axiom:

$$\begin{tikzcd} \bigotimes_{i=1}^{2} \mathfrak{F}_{\mathfrak{A}}(M_i) \arrow[r, "\mathfrak{F}_{\mathfrak{A}}(\underline{f})"] \arrow[d, "\text{permute}"'] & \mathfrak{F}_{\mathfrak{A}}(N) \\ \bigotimes_{i=1}^{2} \mathfrak{F}_{\mathfrak{A}}(M_{\tau(i)}) \arrow[ur, "\mathfrak{F}_{\mathfrak{A}}(\underline{f}\tau)"'] & \end{tikzcd}$$

But this would imply that observables coming from regions that are NOT causally disjoint commute (which is not true in general).

$\mathbf{tPFA}^{\mathrm{add,c}} \to \mathbf{AQFT}^{\mathrm{add},c}$ ... where things get harder

This is where Cauchy constancy and additivity become crucial.

- Let $\mathfrak{F} \in \mathbf{tPFA}^{\mathrm{add,c}}$. We want to define a corresponding $\mathfrak{A}_{\mathfrak{F}} \in \mathbf{AQFT}$. .

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- ◇ **Problem**: $\mathfrak{F}(M)$ is a vector space and not an algebra. We need multiplication maps $\mu_M : \mathfrak{F}(M) \otimes \mathfrak{F}(M) \to \mathfrak{F}(M)$ and we need to prove that with this choices of multiplications:
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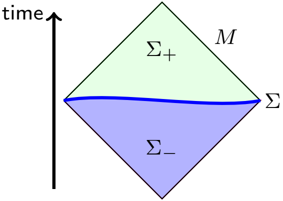
- <ins>*Choice of multiplications:*</ins> Choose Cauchy surface $\Sigma \subset M$, consider chronological future/past part $\Sigma_\pm := I^\pm_M(\Sigma)$ and define via **Cauchy constancy**

$$\begin{tikzcd} \mathfrak{F}(M)\otimes\mathfrak{F}(M) \arrow[rr, "\mu_M"] & & \mathfrak{F}(M) \\ & \mathfrak{F}(\Sigma_+)\otimes\mathfrak{F}(\Sigma_-) \arrow[ul, "\cong", "\mathfrak{F}(\iota^M_{\Sigma_+})\otimes\mathfrak{F}(\iota^M_{\Sigma_-})"'] \arrow[ur, "\mathfrak{F}(\iota^M_{\Sigma})"'] & \end{tikzcd}$$



Multiplication is independent on the choice of Σ

Def: For $M \in \mathbf{Loc}$, denote by $\mathbf{P}_M$ the category of all pairs $U_\pm \subseteq M$ of causally convex open subsets fulfilling the requirements:

(i) there exists a Cauchy surface $\Sigma \subset M$ s.t. $U_\pm \subseteq I^\pm_M(\Sigma)$,

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As a consequence, for every $\mathfrak{F} \in \mathbf{tPFA}^{\mathrm{c}}$ and $M \in \mathbf{Loc}$, the multiplication

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Rem: This step does not yet require the additivity property for $\mathfrak{F}$, but it crucially relies on Cauchy constancy.

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Lem: Let $\mathfrak{F} \in \mathbf{tPFA}^{\mathrm{c}}$ and $f : M \to N$ be $\mathbf{Loc}$-morphism s.t. $f(M) \subseteq N$ is **relatively compact**. Then $\mathfrak{F}(f) : \mathfrak{F}(M) \to \mathfrak{F}(N)$ preserves units and multiplications, i.e. $\mathfrak{F}(f) \circ \eta_M = \eta_N$ and $\mathfrak{F}(f) \circ \mu_M = \mu_N \circ (\mathfrak{F}(f) \otimes \mathfrak{F}(f))$.

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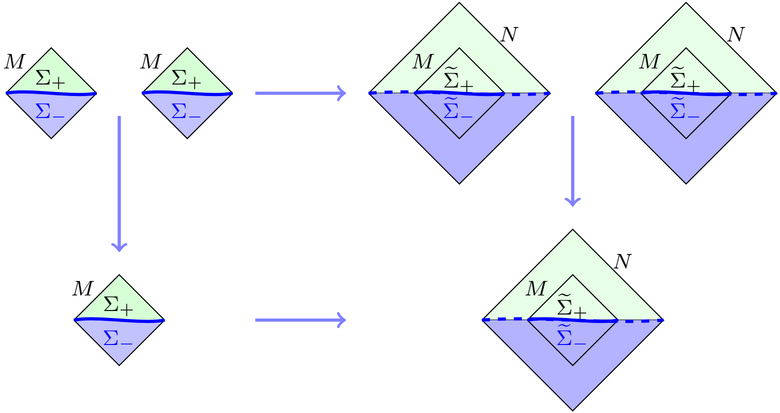
Rem: The proof uses Bernal/Sanchez to extend Cauchy surfaces, hence it relies on the relatively compact assumption.



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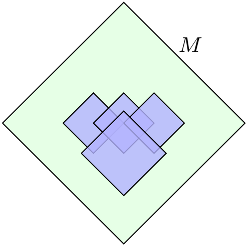
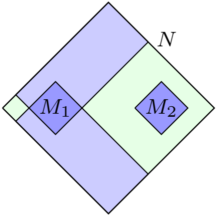


- If $\mathfrak{F} \in \mathbf{tPFA}^{\mathrm{add,c}}$ is also **additive**, $\mathfrak{F}(M) \cong \mathrm{colim}\big(\mathfrak{F}|_M : \mathbf{RC}_M \to \mathbf{Vec}\big)$ is 'generated' from relatively compact subsets, which allows us to prove:

Prop: $\mathfrak{A}_{(-)}$ defines a functor $\mathbf{tPFA}^{\mathrm{add,c}} \to \mathbf{Fun}(\mathbf{Loc}, \mathbf{Alg})$.

(2) Einstein causality

Lem: Let $\mathfrak{F} \in \mathbf{tPFA}^{\mathrm{c}}$ and $(f_1 : M_1 \to N, f_2 : M_2 \to N)$ causally disjoint s.t. both $f_1(M_1), f_2(M_2) \subseteq N$ are **relatively compact**. In this case $\mathfrak{A}_{\mathfrak{F}} : \mathbf{Loc} \to \mathbf{Alg}$ satisfies Einstein causality, i.e. $\mu_N \circ (\mathfrak{F}(f_1) \otimes \mathfrak{F}(f_2)) = \mu_N^{\mathrm{op}} \circ (\mathfrak{F}(f_1) \otimes \mathfrak{F}(f_2))$.



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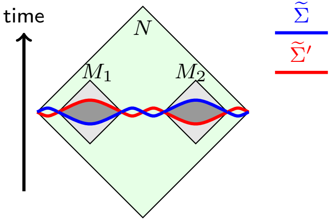
Rem: Relatively compactness is again crucial to extend Cauchy surfaces!

- The key steps to prove Einstein causality are:
  - (1) to find two Cauchy surfaces of N with opposite time-order when restricted to M_1 and M_2
  - (2) use the equivariance axiom.



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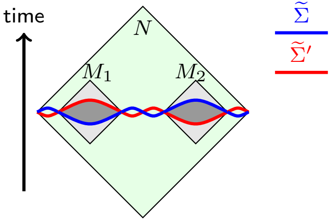

- Using additivity we can then prove that:

Prop: $\mathfrak{A}_{(-)}$ defines a functor $\mathbf{tPFA}^{\mathrm{add,c}} \to \mathbf{AQFT}^{\mathrm{add,c}}$.

Summary of the Main Equivalence Theorem

Theorem (Benini,MP,Schenkel)

The two functors

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described before are inverses of each other. Hence, they define an equivalence (which, to be honest, is an isomorphism)

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between the category of Cauchy constant additive time-orderable prefactorization algebras on $\mathbf{Loc}$ *and the category of Cauchy constant additive AQFTs on* $\mathbf{Loc}$.

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Thanks for your attention!