Renormalization of SU(2) Yang–Mills theory with Flow Equations

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Main features

Historically the first papers on the renormalization flow equations were written by Wilson, Wegner, Polchinski.
In the context of gauge theories this idea was later developed by Reuter, Wetterich; Becchi; Bonini, D'Attanasio, Marchesini; Morris; Fröb, Holland, Hollands...

The main features of the present work

- Bounds on the 1PI functions in momentum space.
- Convergence of 1PI functions in the UV limit.
- Slavnov–Taylor identities in the UV limit.
- Renormalization conditions are imposed at physical points.

Terminology

We define the η-function by

$$\eta(\vec{p}) = \min_{\mathbb{I}\in\wp_{n-1}\setminus\{\emptyset\}} (|\sum_{i\in\mathbb{I}} p_i|, M) \,.$$

where $\wp_{n-1}$ denotes the power set of the index set $\{0, ..., n-1\}$.
A momentum configuration $\vec{p}$ is said nonexceptional iff $\eta(\vec{p}) \neq 0$.

- $[\Gamma^{\vec{\phi}}] < 0$ irrelevant,
- $[\Gamma^{\vec{\phi}}] \geqslant 0$ relevant,
- $[\Gamma^{\vec{\phi}}] = 0$ marginal,
- $[\Gamma^{\vec{\phi}}] > 0$ strictly relevant,

where $[...]$ stands for the mass dimension.

Faddeev–Popov quantization with Lorenz gauge fixing

Semiclassical Lagrangian density

$$\tilde{\mathcal{L}}_0^{tot} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2 - \partial_\mu \bar{c}^a (D_\mu c)^a,$$

$$D_\mu c = \partial_\mu c - ig[A_\mu, c],$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu],$$

$\xi > 0$ is the Feynman parameter.

$F_{\mu\nu}$, c, A_μ are elements of the algebra, e.g. $A_\mu = t^a A^a_\mu$.

$$(t_c)^{ab} = i\epsilon_{abc}, \qquad [t_a, t_b] = i\epsilon_{abc} t_c, \qquad a, b, c \in \{1, 2, 3\}.$$

The counterterms

All counterterms respecting the global symmetries and having ghost number zero

$$\mathcal{L}_{ct}^{\Lambda_0\Lambda_0} = r^{\bar{c}c\bar{c}c}\bar{c}^b c^b \bar{c}^a c^a + r_1^{\bar{c}cAA}\bar{c}^b c^b A_\mu^a A_\mu^a + r_2^{\bar{c}cAA}\bar{c}^a c^b A_\mu^a A_\mu^b$$
$$+ r_1^{A^4} A_\mu^b A_\nu^b A_\mu^a A_\nu^a + r_2^{A^3} A_\nu^b A_\nu^b A_\mu^a A_\mu^a + 2\epsilon_{abc} r^{A^3} (\partial_\mu A_\nu^a) A_\mu^b A_\nu^c$$
$$- r_1^{A\bar{c}c}\epsilon_{abd}(\partial_\mu \bar{c}^a) A_\mu^b c^d - r_2^{A\bar{c}c}\epsilon_{abd}\bar{c}^a A_\mu^b \partial_\mu c^d + \Sigma^{\bar{c}c}\bar{c}^a \partial^2 c^a$$
$$- \frac{1}{2}\Sigma_T^{AA} A_\mu^a(\partial^2\delta_{\mu\nu} - \partial_\mu\partial_\nu)A_\nu^a + \frac{1}{2\xi}\Sigma_L^{AA}(\partial_\mu A_\mu^a)^2$$
$$+ \delta m_{AA}^2 A_\mu^a A_\mu^a + \delta m_{\bar{c}c}^2 \bar{c}^a c^a .$$

There are eleven marginal counterterms which depend on Λ_0.

$$\mathcal{L}_{ct} = \sum_{l>0} \hbar^l \mathcal{L}_{ct;l}$$

The complex measure

Let $d\nu_{\Lambda\Lambda_0}$ be a Gaussian measure with characteristic function

$$\chi^{\Lambda\Lambda_0}(j,b,\bar\eta,\eta) = e^{\frac{1}{\hbar}\langle\bar\eta,S^{\Lambda\Lambda_0}\eta\rangle - \frac{1}{2\hbar}\langle j,C^{\Lambda\Lambda_0}j\rangle - \frac{1}{2\hbar\xi}\langle b,b\rangle}.$$

To obtain the AGE we need an auxiliary field B.

$$d\mu_{\Lambda\Lambda_0}(A,B,c,\bar c) = d\nu_{\Lambda\Lambda_0}(A,B-i\frac{1}{\xi}\partial A,c,\bar c).$$

$$C^{\Lambda\Lambda_0}_{\mu\nu} = \frac{1}{p^2}(\delta_{\mu\nu} + (\xi-1)\frac{p_\mu p_\nu}{p^2})\,\sigma_{\Lambda\Lambda_0}(p^2)\,,$$

$$S^{\Lambda\Lambda_0}(p) = \frac{1}{p^2}\,\sigma_{\Lambda\Lambda_0}(p^2)\,,\quad \sigma_{\Lambda\Lambda_0}(p^2) = e^{-\frac{p^4}{\Lambda_0^4}} - e^{-\frac{p^4}{\Lambda^4}}\,.$$

Λ, Λ_0 are IR and UV cutoffs, $0 < \Lambda \leqslant \Lambda_0$.

Generating functionals

Partition function of Yang–Mills theory

$$Z^{\Lambda\Lambda_0}(K) = \int d\mu_{\Lambda\Lambda_0}(\Phi)\, e^{-\frac{1}{\hbar}L^{\Lambda_0\Lambda_0}}\, e^{\frac{1}{\hbar}\langle K,\Phi\rangle}.$$

$$\begin{aligned}\mathcal{L}^{\Lambda_0\Lambda_0} &= g\epsilon_{abc}\partial_\mu A_\nu^a A_\mu^b A_\nu^c + \frac{g^2}{4}\epsilon_{cab}\epsilon_{cds}A_\mu^a A_\nu^b A_\mu^d A_\nu^s \\ &\quad - g\epsilon_{abc}\partial_\mu \bar{c}^a A_\mu^b c^c + \mathcal{L}_{ct}^{\Lambda_0\Lambda_0}\,.\end{aligned}$$

The tree level interaction $\mathcal{L}_0^{\Lambda_0\Lambda_0}$ does not depend on the B field.
Generating functional of the Connected Schwinger (CS) functions with regulator $\sigma_{\Lambda\Lambda_0}$

$$W^{\Lambda\Lambda_0} = \hbar \log Z^{\Lambda\Lambda_0}.$$

Generating functional of the Connected Amputated Schwinger (CAS) functions

$$L^{\Lambda\Lambda_0}(\Phi) = -\hbar \log \int d\mu_{\Lambda\Lambda_0}(\Phi')\, e^{-\frac{1}{\hbar}L^{\Lambda_0\Lambda_0}(\Phi'+\Phi)}$$

The BRST symmetry

The full tree level Lagrangian density in the limit $\Lambda \to 0$, $\Lambda_0 \to \infty$

$$\mathcal{L}_0^{tot} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{\xi}{2} B^2 - iB\partial_\mu A_\mu - \partial_\mu \bar{c} D_\mu c \, .$$

is invariant under the infinitesimal BRST transformation

$$\delta^{BRS} A = \epsilon \, Dc, \qquad \delta^{BRS} c = \epsilon \, \frac{1}{2} ig\{c, c\},$$
$$\delta^{BRS} \bar{c} = \epsilon \, iB, \qquad \delta^{BRS} B = 0,$$

where ϵ is a Grassmann parameter, and $\{c, c\}^d = i\epsilon_{abd} c^a c^b$. Defining the classical operator s

$$\delta^{BRS} \Phi = \epsilon \, s \, \Phi, \qquad s^2 = 0.$$

BRST invariance is explicitly broken by the regulators.

The effective action

The effective action is the Legendre transform of W

$$\Gamma^{\Lambda\Lambda_0}(\Phi) = \langle K, \Phi \rangle - W^{\Lambda\Lambda_0}(K)\,.$$

Reduced effective action

$$\mathsf{\Gamma}^{\Lambda\Lambda_0}(\Phi) = \Gamma^{\Lambda\Lambda_0}(\Phi) - \frac{1}{2}\langle \Phi, \mathbf{C}^{-1}_{\Lambda\Lambda_0}\Phi \rangle$$

By definition in the expansion over $\hbar$

$$\mathsf{\Gamma} = \sum_{l=0}^{\infty} \hbar^l \mathsf{\Gamma}_l \qquad\qquad \mathsf{\Gamma}_{l=0}^{\Lambda\Lambda_0;\phi\phi} = 0$$

Flow equation

Wilson Wigner Polchinski...

$$\dot{L} = \frac{\hbar}{2}\langle \frac{\delta}{\delta\Phi}, \dot{\mathbf{C}}\frac{\delta}{\delta\Phi}\rangle L - \frac{1}{2}\langle \frac{\delta L}{\delta\Phi}, \dot{\mathbf{C}}\frac{\delta L}{\delta\Phi}\rangle \qquad \dot{L} := \partial_\Lambda L^{\Lambda\Lambda_0}$$

Wetterich, Bonini, D'Attanasio, Marchesini, Morris...

$$\dot{\Gamma} = \frac{\hbar}{2}\langle \dot{\mathbf{C}}\, \delta_\phi \delta_{\bar{\phi}} \Gamma \sum_{m=0}^{\infty} \left(-\mathbf{C}\, \delta_{\bar{\phi}} \delta_\phi \Gamma\right)^m \rangle \qquad \dot{\Gamma} := \partial_\Lambda \Gamma^{\Lambda\Lambda_0}$$

Reconstruction

We proceed by induction in the loop order l: $\Gamma = \sum_{l=0}^{\infty} \hbar^l \Gamma_l$.

The irrelevant terms are constructed by integrating the FE from Λ_0 down to Λ.

$$\Gamma_l^{\Lambda\Lambda_0;\vec{\phi};w} = \Gamma_l^{\Lambda_0\Lambda_0;\vec{\phi};w} + \int_{\Lambda_0}^{\Lambda} d\lambda\, \dot{\Gamma}_l^{\lambda\Lambda_0;\vec{\phi};w}, \quad \Gamma_l^{\Lambda_0\Lambda_0;\vec{\phi};w} = 0$$

$$\dot{\Gamma}^{\vec{\phi}} = \frac{\hbar}{2} \sum_{\vec{\Phi}=(\vec{\phi}_1,\ldots)} \langle \dot{\mathbf{C}}_{\zeta\bar{\zeta}} \mathcal{F}^{\zeta\vec{\Phi}\bar{\zeta}} \rangle$$

$$\mathcal{F}^{\zeta_1\vec{\Phi}\bar{\zeta}_m} = \Gamma^{\zeta_1\vec{\phi}_1\bar{\zeta}_1} \prod_{j=2}^{m} \mathbf{C}_{\zeta_j\bar{\zeta}_{j-1}} \Gamma^{\zeta_j\vec{\phi}_j\bar{\zeta}_j}, \quad \Gamma^{\zeta\bar{\zeta}}_{l=0} = 0$$

Here $\delta(\sum_{i=0}^{n-1} p_i)\Gamma^{\vec{\phi};w}(\vec{p}) := \partial^w \Big(\prod_{i=0}^{n-1} \frac{\delta}{\delta\phi_i(p_i)} \Big) \Gamma \Big|_{\vec{\phi}=0}$

For the relevant terms we integrate the FE from $\Lambda = 0$ up to arbitrary Λ at the renormalization point $\vec{q}$

$$\Gamma_l^{\Lambda\Lambda_0;\vec{\phi};w}(\vec{q}) = \Gamma_l^{0\Lambda_0;\vec{\phi};w}(\vec{q}) + \int_0^\Lambda d\lambda\, \dot{\Gamma}_l^{\lambda\Lambda_0;\vec{\phi};w}(\vec{q})$$

We interpolate this to arbitrary momenta $\vec{p}$ integrating over the corresponding irrelevant terms;

$$\Gamma_l^{\Lambda\Lambda_0;\vec{\phi};w}(\vec{p}) = \Gamma_l^{\Lambda\Lambda_0;\vec{\phi};w}(\vec{q}) + \int_{\vec{q}}^{\vec{p}} \partial\Gamma_l^{\Lambda\Lambda_0;\vec{\phi};w}$$

We need bounds to prove the existence of $\lim_{\substack{\Lambda\to 0,\\ \Lambda_0\to\infty}} \Gamma^{\Lambda\Lambda_0;\vec{\phi}}$.

Trivial dependence on the B field

Vanishing renormalization conditions for all relevant terms

$$\partial^w \Gamma_l^{0\Lambda_0;B\vec{\phi}}(\vec{q}) = 0\,,$$

imply that $\forall \Lambda > 0$

$$\Gamma_l^{\Lambda\Lambda_0;B\vec{\phi}}(\vec{p}) = 0\,.$$

No counterterms with the field B. Dependence on the B-fields known explicitly:

$$\Gamma^{\Lambda\Lambda_0}(A,B,c,\bar{c}) = \frac{1}{2\xi}\int d^4x\,(\xi B - i\partial A)^2 + \tilde{\Gamma}^{\Lambda\Lambda_0}(A,c,\bar{c})\,.$$

Violated Slavnov–Taylor Identities (VSTI)

Introduce the Lagrangian density

$$\mathcal{L}_{vst}^{\Lambda_0\Lambda_0} = \mathcal{L}^{\Lambda_0\Lambda_0} + \gamma\psi^{\Lambda_0} + \omega\Omega^{\Lambda_0},$$

where γ, ω are external sources, $R_i^{\Lambda_0} = 1 + O(\hbar)$,

$$\psi^{\Lambda_0} = R_1^{\Lambda_0}\partial c - igR_2^{\Lambda_0}[A, c], \qquad \Omega^{\Lambda_0} = \frac{1}{2i}gR_3^{\Lambda_0}\{c, c\}\,.$$

We make the change of variables $\Phi \mapsto \Phi + \delta_\epsilon\Phi$

$$Z^{0\Lambda_0}(K) = \int d\mu_{0\Lambda_0}(\Phi)\, e^{-\frac{1}{\hbar}L_{vst}^{\Lambda_0\Lambda_0}}\; e^{\frac{1}{\hbar}\langle K,\Phi\rangle}.$$

$$\delta_\epsilon A = \epsilon\,\sigma_{0\Lambda_0}\psi^{\Lambda_0}, \qquad \delta_\epsilon c = -\epsilon\,\sigma_{0\Lambda_0}\Omega^{\Lambda_0}, \qquad \delta_\epsilon \bar{c} = \epsilon\,\sigma_{0\Lambda_0} iB\,.$$

Performing the Legendre transform the identity $\delta_\epsilon Z^{0\Lambda_0} = 0$ gives

$$\Gamma_1^{0\Lambda_0} + \int d^4x \, (iB + \frac{1}{\xi}\partial A)\Gamma_\beta^{0\Lambda_0} = \frac{1}{2}\mathcal{S}\underline{\Gamma}^{0\Lambda_0},$$

where we have introduced an auxiliary functional $\underline{\Gamma}^{0\Lambda_0}$

$$\underline{\Gamma}^{0\Lambda_0} = i\langle B, \bar{\omega}\rangle + \underline{\tilde{\Gamma}}^{0\Lambda_0}, \qquad \underline{\tilde{\Gamma}}^{0\Lambda_0} = \tilde{\Gamma}^{0\Lambda_0} + \frac{1}{2\xi}\langle A, \partial\partial\, A\rangle\,,$$

and $\mathcal{S} = \mathcal{S}_{\tilde{c}} + \mathcal{S}_A - \mathcal{S}_c$, with

$$\mathcal{S}_\phi = \langle \frac{\delta \underline{\Gamma}^{0\Lambda_0}}{\delta\phi}, \sigma_{0\Lambda_0}\frac{\delta}{\delta\phi^\ddagger}\rangle + \langle \frac{\delta \underline{\Gamma}^{0\Lambda_0}}{\delta\phi^\ddagger}, \sigma_{0\Lambda_0}\frac{\delta}{\delta\phi}\rangle,$$

$$(\phi, \phi^\ddagger) \in \{(A,\gamma), (c,\omega), (\tilde{c},\bar{\omega})\}, \quad \frac{\delta}{\delta\tilde{c}} = \frac{\delta}{\delta\bar{c}} - \partial\frac{\delta}{\delta\gamma}\,.$$

Nilpotency of S

We rewrite equation VSTI in the following form

$$\langle iB, \tilde{\Gamma}^{0\Lambda_0}_{\beta}\rangle = \frac{1}{2}\mathcal{S}_{\tilde{c}}\underline{\Gamma}^{0\Lambda_0} = \langle iB, \sigma_{0\Lambda_0}\frac{\delta}{\delta\tilde{c}}\underline{\tilde{\Gamma}}^{0\Lambda_0}\rangle,$$
$$\tilde{\mathrm{F}}^{0\Lambda_0}_{1} = \frac{1}{2}\tilde{S}\underline{\tilde{\Gamma}}^{0\Lambda_0},$$

where

$$\tilde{S} = \mathcal{S}_A - \mathcal{S}_c, \qquad \tilde{\mathrm{F}}^{0\Lambda_0}_{1} = \tilde{\Gamma}^{0\Lambda_0}_{1} + \frac{1}{\xi}\langle\partial A, \tilde{\Gamma}^{0\Lambda_0}_{\beta}\rangle\,.$$

Important properties of $\mathcal{S}_\phi$: $\forall\phi,\phi'\in\{A,c,\tilde{c}\}$

$$(\mathcal{S}_\phi\mathcal{S}_{\phi'} + \mathcal{S}_{\phi'}\mathcal{S}_\phi)\underline{\Gamma}^{0\Lambda_0} = 0.$$

It follows that $\tilde{S}^2\underline{\tilde{\Gamma}}^{0\Lambda_0} = 0$, $\tilde{S}\mathcal{S}_{\tilde{c}} = -\mathcal{S}_{\tilde{c}}\tilde{S}$ and consequently

$$\tilde{S}\tilde{\mathrm{F}}_1 = 0, \qquad \tilde{S}\tilde{\Gamma}_{\beta} + \sigma_{0\Lambda_0}\left(\frac{\delta}{\delta\bar{c}} - \partial\frac{\delta}{\delta\gamma}\right)\tilde{\mathrm{F}}_1 = 0.$$

Explicit form of the VSTI

$$\tilde{\Gamma}_{\beta}^{0\Lambda_0} = \sigma_{0\Lambda_0}\left(\frac{\delta\tilde{\underline{\Gamma}}^{0\Lambda_0}}{\delta\bar{c}} - \partial\tilde{\underline{\Gamma}}_{\gamma}^{0\Lambda_0}\right) \qquad (VAGE),$$

$$\tilde{\mathrm{F}}_{1}^{0\Lambda_0} = \langle\frac{\delta\tilde{\underline{\Gamma}}^{0\Lambda_0}}{\delta A}, \sigma_{0\Lambda_0}\tilde{\underline{\Gamma}}_{\gamma}^{0\Lambda_0}\rangle - \langle\frac{\delta\tilde{\underline{\Gamma}}^{0\Lambda_0}}{\delta c}, \sigma_{0\Lambda_0}\tilde{\underline{\Gamma}}_{\omega}^{0\Lambda_0}\rangle \qquad (VSTI).$$

The goal is to show that $\tilde{\Gamma}_{\beta}^{0\infty} = 0$, $\tilde{\mathrm{F}}_{1}^{0\infty} = 0$ which imply $\tilde{\Gamma}_{1}^{0\infty} = 0$.

Another form of the VSTI

$$\begin{aligned}\tilde{\Gamma}_{1}^{0\Lambda_0} =& \langle\frac{\delta\tilde{\Gamma}^{0\Lambda_0}}{\delta A}, \sigma_{0\Lambda_0}\tilde{\Gamma}_{\gamma}^{0\Lambda_0}\rangle - \langle\frac{\delta\tilde{\Gamma}^{0\Lambda_0}}{\delta c}, \sigma_{0\Lambda_0}\tilde{\Gamma}_{\omega}^{0\Lambda_0}\rangle \\ & - \frac{1}{\xi}\langle\partial A, \sigma_{0\Lambda_0}\frac{\delta\tilde{\Gamma}^{0\Lambda_0}}{\delta\bar{c}}\rangle .\end{aligned}$$

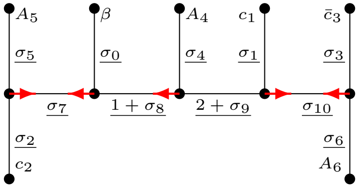
The vertex functions on the rhs gives renormalization conditions for the anomalies on the lhs.

Renormalization conditions

- $\partial^w \Gamma^{0\Lambda_0;\vec{\phi}}_{\vec{\varkappa}}(0) = 0$, for all strictly relevant terms, $\varkappa \in \{\gamma, \omega\}$.
- $\Gamma^{M\Lambda_0;c\bar{c}c\bar{c}}(0) = 0$, $\Gamma^{M\Lambda_0;c\bar{c}A^2}(0) = 0$, $\partial_A \Gamma^{M\Lambda_0;c\bar{c}A}(0) = 0$, We show that the counterterms $r^{\bar{c}c\bar{c}c}$, $r_1^{\bar{c}cA^2}$, $r_2^{\bar{c}cA^2}$, $r_2^{A\bar{c}c}$ vanish.
- The renormalization constants r^{A^3} , Σ_T^{AA} , $\Sigma^{\bar{c}c}$ are free.
- The remaining renormalization constants must satisfy 7 additional relations in order to make the marginal terms $\Gamma_1^{\vec{\phi};w}$, $\Gamma_\beta^{\vec{\phi};w}$ at the renormalization point comply with the bounds **3** and **4** below.
- We prove the existence of a solution for this system of relations that does not depend on the UV cutoff.

Trees $\mathcal{T}_{\vec{\varphi}}$ and $\mathcal{T}_{1\vec{\varphi}}$

- Vertices of valence one and three, $V = V_1 \cup V_3$.
- An edge e carries momentum p_e. Momentum conservation at the vertices.
- To any edge are associated $\rho(e) \in \{0, 1, 2\}$ and the number of momentum derivatives $\sigma(e)$.
- An edge has the θ-weight, $\theta(e) = \rho(e) + \sigma(e)$.



$$\chi : V_\bullet \to E \backslash E_1$$

$$\rho(e) = 2 - |\chi^{-1}(e)|$$

The main elements of the bounds

For a tree τ we sum over the family of θ-weights $\Theta_\tau^w = \{\theta_j(e)\}_j$.

$$\Pi^{\Lambda}_{\tau,\theta}(\vec{p}) = \prod_{e\in E} \frac{1}{(\Lambda + |p_e|)^{\theta(e)}} \,,$$

$$Q^{\Lambda;w}_{\tau}(\vec{p}) = \begin{cases} \inf_{i\in\mathbb{I}} \sum_{\Theta_\tau^{w'(i)}} \Pi^{\Lambda}_{\tau,\theta}(\vec{p}), & |V_1| = 3\,, \\ \sum_{\Theta_\tau^{w}} \Pi^{\Lambda}_{\tau,\theta}(\vec{p}), & otherwise\,. \end{cases}$$

$w'(i)$ is obtained from w by diminishing w_i by one unit, and, for nonvanishing w, $\mathbb{I} = \{i : w_i > 0\}$.

Bounds on vertex functions

Let $d = 4 - 2n_{\varkappa} - \mathrm{N} - \|w\|$.

1.a) $d \geqslant 0$ or $\mathrm{N} + n_{\varkappa} = 2$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}(\vec{p})| \leqslant (\Lambda + |\vec{p}|)^d P^{\Lambda\Lambda}_r(\vec{p}) \,,$$

1.b) $d < 0$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}(\vec{p})| \leqslant \sum_{\tau \in \mathcal{T}_{\vec{\varkappa}\vec{\phi}}} Q^{\Lambda;w}_{\tau}(\vec{p})\; P^{\Lambda\Lambda}_r(\vec{p}) \,,$$

$$P^{\Lambda\Lambda'}_r(\vec{p}) = \mathcal{P}^{(0)}_r\left(\log_+ \frac{\max(|\vec{p}|, M)}{\Lambda + \eta(\vec{p})}\right) + \mathcal{P}^{(1)}_r\left(\log_+ \frac{\Lambda'}{M}\right).$$

$\mathcal{P}_r$ denotes polynomials with nonnegative coefficients and degree

$$r = \begin{cases} 2l & d \geqslant 0 \\ 2l - 1 & d < 0 \end{cases}$$

Convergence

Let $d = 4 - 2n_{\varkappa} - \mathrm{N} - \|w\|$.

2.a) $d \geqslant 0$ or $\mathrm{N} + n_{\varkappa} = 2$

$$|\partial_{\Lambda_0} \Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}(\vec{p})| \leqslant \frac{\Lambda + M + |\vec{p}|}{\Lambda_0^2} (\Lambda + |p|)^d P_r^{\Lambda\Lambda_0}(\vec{p})\,,$$

2.b) $d < 0$

$$|\partial_{\Lambda_0} \Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}(\vec{p})| \leqslant \frac{\Lambda + M + |\vec{p}|}{\Lambda_0^2} \sum_{\tau \in \mathcal{T}_{\vec{\varkappa}\vec{\phi}}} Q_\tau^{\Lambda;w}(\vec{p})\, P_r^{\Lambda\Lambda_0}(\vec{p})\,.$$

Cauchy criterion

$$|\Gamma^{\Lambda\Lambda_0';\vec{\phi};w}_{\vec{\varkappa};l} - \Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}| \leqslant \int_{\Lambda_0}^{\Lambda_0'} d\lambda_0\, |\partial_{\lambda_0} \Gamma^{\Lambda\lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}|\,.$$

The restoration of AGE

Let $d = 3 - 2n_{\varkappa} - \mathrm{N} - \|w\|$.

3.a) $d \geqslant 0$ or $\mathrm{N} + n_{\varkappa} = 1$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\beta\vec{\varkappa};l}(\vec{p})| \leqslant \frac{M + |\vec{p}| + \Lambda}{\Lambda_0} (\Lambda + |\vec{p}|)^d P^{\Lambda\Lambda_0}_{r\,0}(\vec{p}) \,,$$

3.b) $d < 0$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\beta\vec{\varkappa};l}(\vec{p})| \leqslant \frac{M + |\vec{p}| + \Lambda}{\Lambda_0} \sum_{\tau \in \mathcal{T}_{\beta\vec{\varkappa}\vec{\phi}}} Q^{\Lambda;w}_{\tau}(\vec{p})\, P^{\Lambda\Lambda_0}_{r\,0}(\vec{p}) \,,$$

$$P^{\Lambda\Lambda_0}_{r\,s}(\vec{p}) = \left(1 + \left(\frac{|\vec{p}|}{\Lambda_0}\right)^4\right) \mathcal{P}^{(2)}_s\left(\frac{|\vec{p}|}{\Lambda + M}\right) P^{\Lambda\Lambda_0}_r(\vec{p}) \,.$$

The restoration of STI

Let $d = 5 - 2n_\varkappa - \mathrm{N} - \|w\|$.

4.a) $d > 0$ or $\mathrm{N} + n_\varkappa = 2$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{1\vec{\varkappa};l}(\vec{p})| \leqslant \frac{M + |\vec{p}| + \Lambda}{\Lambda_0}(\Lambda + |\vec{p}|)^d P^{\Lambda\Lambda_0}_{r_1\,s_1}(\vec{p})\,,$$

4.b) $d \leqslant 0$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{1\vec{\varkappa};l}(\vec{p})| \leqslant \frac{M + |\vec{p}| + \Lambda}{\Lambda_0} \sum_{\tau \in \mathcal{T}_{1\vec{\varkappa}\vec{\phi}}} Q^{\Lambda;w}_{\tau}(\vec{p})\, P^{\Lambda\Lambda_0}_{r_1\,s_1}(\vec{p})\,.$$

r_1, s_1 are linear functions of loop number l.

Thank you!

https://arxiv.org/abs/1704.06799