Perturbative construction of interactions involving string-local bosonic potentials

Mathematics of interacting QFT models

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String-local fields

[Mund,Schroer,Yngvason – 2005; Mund,Rehren,Schroer – 2017]

Definition (helicity $s = 1$, higher helicities straight forward)

Let e be a spacelike direction, $e^2 = -1$, $F^{\mu\nu}(x)$ the usual field strength tensor and define

$$A^\mu(x, e) := I_e F^{\mu\nu}(x) e_\nu,$$

where I_e is the string-integral operator $I_e f(x) := \int_0^\infty d\lambda \; f(x + \lambda e)$.

This talk: Massless fields only! (massive case is similar but employs more fields that decouple from the main field in the massless limit)

Advantages:

- Have $\partial_\mu A^\mu = 0$, $e_\mu A^\mu = 0 \Rightarrow$ correct number of d.o.f.
- String-local A^μ lives on physical Hilbert space.
- No ghosts, no BRST needed!
- Stringlocal fields of any helicity have UV-dimension $d_{\mathrm{UV}} = 1$ (but the delocalization increases the renormalization freedom accordingly).

Two-point functions

$$\langle\!\langle A_\mu(x,e)A_\varrho(x',e')\rangle\!\rangle = \int d\mu_0(p)\; e^{-ip(x-x')}\left(-\eta_{\mu\varrho} + \frac{e_\varrho p_\mu}{(pe)_-} + \frac{e'_\mu p_\varrho}{(pe')_+} - \frac{(ee')p_\mu p_\varrho}{(pe)_-(pe')_+}\right)$$

- Additional denominators potentially cause infrared issues
  $\rightsquigarrow$ This talk: concentrate on ultraviolet region. We are safe as long as we smear in test functions $f(x)$ with $\hat{f}(0)=0$.
  Remark: There are cases with and cases without infrared issues!
- 2pf is a distribution on $(\mathbb{R}^4 \times H_{-1})^2$
  $\rightsquigarrow$ subtle notion of scaling behavior: affects renormalization freedom!

Naive choice of the time-ordered two-point function would be

$$\langle\!\langle T_0\, A_\mu(x,e)\, A_\varrho(x',e')\rangle\!\rangle = \int d^4p \frac{e^{-ip(x-x')}}{p^2+i0}\left(-\eta_{\mu\varrho} + \frac{e_\varrho p_\mu}{(pe)_-} + \frac{e'_\mu p_\varrho}{(pe')_+} - \frac{(ee')p_\mu p_\varrho}{(pe)_-(pe')_+}\right),$$

and this is unique up to certain "stringy δ-distributions".

This T_0 is always one choice but it is not clear if it's a good choice!

Stringy δ-distributions:

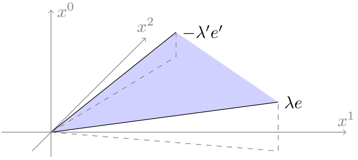
We can add terms $\sim p^2$ to the 2pf kernel without changing the 2pf. But the propagator will change by a linear combination of terms

- $\sim \delta^{(a)}(x - x')$ (point-local),
- $\sim \int_0^\infty d\lambda \int_0^\infty d\lambda' \, u(\lambda, \lambda') \, \delta^{(a)}(x - x' + \lambda e - \lambda e')$ (string-local).

Rigorous formulation (Epstein-Glaser): *splitting of distributions*.
$\rightsquigarrow$ Restrict number of allowed terms by upper bound on scaling behavior:

Scaling degree everywhere determined by

- the number of derivatives, $|a|$,
- the respective codimension,
- and by properties of
  - $u(0,0)$ at $\{(\lambda, \lambda') = (0,0)\}$,
  - $u(\lambda, 0)$ at $\{\lambda \neq 0, \lambda' = 0\}$.



Example: $u(\lambda, \lambda') = \lambda^{n-1} \lambda'^{m-1} \Rightarrow I_e^n I'^m_{e'} \delta(x - x')$

In general: huge renormalization freedom due to presence of $u(\lambda, \lambda')$.

Way out: "Induce" renormalization from $F^{\mu\nu}(x)$

Define T-products for field strengths and lift to potentials: Simplest case would be

$$\langle\langle T A^\mu(x,e) A^\nu(x',e')\rangle\rangle = I_e I'_{e'} \langle\langle T F^{\mu\kappa}(x) F^{\nu\lambda}(x')\rangle\rangle e_\kappa e'_\lambda.$$

Advantages of induction from field strength:

- Renormalization reduces to extension of distributions to the diagonal.
- Function $u(\lambda, \lambda')$ is fixed by renormalization of $F^{\mu\nu}(x)$.

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Obstruction:

Require that time-ordering maps 0 to 0. Specifically,

$$0 = \langle\!\langle T \; (\partial^\kappa F^{\mu\nu} + \text{cyclic}) \; X' \rangle\!\rangle = \langle\!\langle T \, \partial_\mu F^{\mu\nu} \; X' \rangle\!\rangle = \langle\!\langle T \; (dA - F) \; X' \rangle\!\rangle \,,$$

etc. This is **not compatible** with $\langle\!\langle T \, \partial^\mu A^\nu \; X' \rangle\!\rangle \overset{!?}{=} I_e \, \langle\!\langle T \, \partial^\mu F^{\nu\alpha} \; X' \rangle\!\rangle \, e_\alpha$.

$\rightsquigarrow$ If induction is possible, then not in a naive sense!

Brouder-Dütsch-Fredenhagen (BDF) on-shell formalism
[Dütsch,Fredenhagen – 2004; Brouder,Dütsch – 2008]

1. Introduce off-shell algebra $\mathcal{A}_{\text{off}}$ of fields without any relations.
2. Specify on-shell relations on the subspace $\mathcal{A}^{(1)} \subset \mathcal{A}_{\text{off}}$ spanned by linear expressions in the fields and their derivatives.
3. Let I be the ideal in $\mathcal{A}_{\text{off}}$ generated by these relations.
4. Define on-shell algebra $\mathcal{A}_{\text{on}} := \mathcal{A}_{\text{off}}/\sim$, where $A \sim B \Leftrightarrow A - B \in I$.
5. $\pi : \mathcal{A}_{\text{off}} \to \mathcal{A}_{\text{on}}$ the canonical projection and $\xi : \mathcal{A}_{\text{on}} \to \mathcal{A}_{\text{off}}$ an algebra homomorphism – picking a representative – s.t.
   - $\pi\xi = \text{id}$,
   - $\xi\pi(\mathcal{A}^{(1)}) \subset \mathcal{A}^{(1)}$,
   - $\xi\pi$ does not worsen the scaling behavior of the fields,
   - Lorentz transformations commute with $\xi\pi$.

$\rightsquigarrow$ in many cases, ξ is uniquely fixed!

Brouder-Dütsch-Fredenhagen (BDF) on-shell formalism

On-shell time-ordered products

Given *off*-shell fields $\phi_1, \ldots, \phi_n$, define the *on*-shell time-ordered product

$$T^{\text{on}}\left(\pi(\phi_1), \ldots, \pi(\phi_n)\right) = T^{\text{off}}\left(\xi\pi(\phi_1), \ldots, \xi\pi(\phi_n)\right),$$

where T^{off} commutes with derivatives and satisfies the usual requirements for time-ordering:
linearity, symmetry in the all arguments, causal factorization, $T^{\text{off}}(\phi) = \phi$.

- No derivatives: $\langle\!\langle T^{\text{on}}\, \pi(\phi_1)\, \pi(\phi_2) \rangle\!\rangle$ may have renorm. freedom,
- but since T^{off} commutes with derivatives, no new renormalization constants appear in $\langle\!\langle T^{\text{on}}\, \pi(\partial^\alpha\phi_1)\, \pi(\partial^\beta\phi_2) \rangle\!\rangle$!

$\rightsquigarrow$ Possible interference between BDF and Epstein-Glaser constructions.

Adjust BDF to string-local fields to achieve “induced renormalization”.

Example: massless vector field

- View point-local field strength as fundamental,
- write $A^\mu = I_e F^{\mu\nu} e_\nu$ (as a kind of shorthand notation),
- algebra generated by $F^{\mu\nu}$, $I_e^n F^{\mu\nu} e_\nu$ and their derivatives,
- ideal is generated by

$$I_1 := \partial^\kappa F^{\mu\nu} + \text{cyclic}, \qquad I_2 := \partial_\mu F^{\mu\nu}.$$

Start with fields with no derivatives:

$$\begin{aligned} \xi\pi(F^{\mu\nu}) &= F^{\mu\nu}, \\ \xi\pi(A^\mu) &= \xi\pi(I_e F^{\mu\nu} e_\nu) = I_e F^{\mu\nu} e_\nu = A^\mu, \\ \xi\pi(\partial^\kappa F^{\mu\nu}) &= \partial^\kappa F^{\mu\nu} - \frac{1}{3}\{(\partial^\kappa F^{\mu\nu} + \text{cyclic}) + \eta^{\mu\kappa}\partial_\varrho F^{\varrho\nu} - \eta^{\nu\kappa}\partial_\varrho F^{\varrho\mu}\} \end{aligned}$$

Example: massless vector field

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$$\xi\pi(A^{\mu}) = \xi\pi(I_e F^{\mu\nu} e_\nu) = I_e F^{\mu\nu} e_\nu = A^\mu,$$
$$\xi\pi(\partial^\kappa F^{\mu\nu}) = \partial^\kappa F^{\mu\nu} - \frac{1}{3}\left\{(\partial^\kappa F^{\mu\nu} + \text{cyclic}) + \eta^{\mu\kappa}\partial_\varrho F^{\varrho\nu} - \eta^{\nu\kappa}\partial_\varrho F^{\varrho\mu}\right\}.$$

$$\xi\pi(\partial^\kappa A^\mu) = \partial^\kappa A^\mu + d_1 I_e \left\{\partial^\kappa F^{\mu\nu} + \text{cyclic}\right\} e_\nu + d_2 I_e \left\{\left(\eta^{\mu\kappa} - \frac{e^\mu e^\kappa}{e^2}\right)\partial_\varrho F^{\varrho\nu}\right\} e_\nu$$

- $\eta_{\kappa\mu}\, \xi\pi(\partial^\kappa A^\mu) \overset{!}{=} 0 \quad \Rightarrow \quad d_2 = -\frac{1}{3}$,
- $\xi\pi(\partial^\kappa A^\mu - \partial^\mu A^\kappa) \overset{!}{=} \xi\pi(F^{\kappa\mu}) = F^{\kappa\mu} \quad \Rightarrow \quad d_1 = -\frac{1}{2}$,
- $e_\mu\, \xi\pi(\partial^\kappa A^\mu) = 0$ and $e_\kappa\, \xi\pi(\partial^\kappa A^\mu) = -F^{\mu\nu} e_\nu$ (by choice of "axial combination" in d_2-term).

Example: massless vector field

$$\xi\pi(F^{\mu\nu}) = F^{\mu\nu},$$

$$\xi\pi(A^{\mu}) = \xi\pi(I_e F^{\mu\nu} e_\nu) = I_e F^{\mu\nu} e_\nu = A^\mu,$$

$$\xi\pi(\partial^\kappa F^{\mu\nu}) = \partial^\kappa F^{\mu\nu} - \frac{1}{3}\left\{(\partial^\kappa F^{\mu\nu} + \text{cyclic}) + \eta^{\mu\kappa}\partial_\varrho F^{\varrho\nu} - \eta^{\nu\kappa}\partial_\varrho F^{\varrho\mu}\right\},$$

$$\xi\pi(\partial^\kappa A^{\mu}) = \partial^\kappa A^\mu - I_e\left\{\frac{1}{2}(\partial^\kappa F^{\mu\nu} + \text{cyclic}) + \frac{1}{3}\left(\eta^{\mu\kappa} - \frac{e^\mu e^\kappa}{e^2}\right)\partial_\varrho F^{\varrho\nu}\right\} e_\nu .$$

Conclusion:

Induction of string-local time-ordering from point-local field strength is possible, but not in a naive way:

$$\xi\pi(\partial^\kappa A^\mu) = \xi\pi(I_e \partial^\kappa F^{\mu\nu} e_\nu) \neq I_e \xi\pi(\partial^\kappa F^{\mu\nu}) e_\nu$$

but $\xi\pi(\partial^\kappa A^\mu) = I_e \xi\pi(\text{some point-local } X^\nu) e_\nu$.

A string-local Epstein-Glaser programme

Construct the scattering matrix in the string-local setting:

$$S(g,h) = 1 + \sum_{n=1}^{\infty} \frac{i^n}{n!} \int d^{4n}x \, d^n\sigma(\underline{e}) \, T\left[\mathcal{L}_{\text{int}}(x_1, \underline{e}_1) \cdots \mathcal{L}_{\text{int}}(x_n, \underline{e}_n)\right]$$
$$\times \prod_{i=1}^{n} g(x_i) \prod_{j} h(e_{i,j}).$$

$S(g,h)$ should be independent of the choice of h:

$$\frac{\delta S(g,h)}{\delta h} \stackrel{!}{=} 0.$$

This is satisfied in the adiabatic limit wrt x if

Perturbative string independence (SI)

$$d_{e_{i,j}} T\left[\mathcal{L}_{\text{int}}(x_1, \underline{e}_1) \cdots\right] = \frac{\partial}{\partial x_i^{\mu}} T\left[\mathcal{L}_{\text{int}}(x_1, \underline{e}_1) \cdots Q^{\mu}(x_i, \underline{e}_i) \cdots\right] \quad \forall i, j, n.$$

At lowest non-trivial order:

$$T\left[\mathcal{L}_{\mathsf{int}}(x,\underline{e})\right] = \mathcal{L}_{\mathsf{int}}(x,\underline{e}) \quad \Rightarrow$$

L-Q-pair

$$d_{e_i^\kappa}\mathcal{L}_{\mathsf{int}}(x,\underline{e}) \stackrel{!}{=} \partial_\mu Q^\mu_\kappa(x,\underline{e})$$

At higher orders:

- Can we exploit renormalization freedom to achieve SI? $\leftrightarrow$ Does SI fix/reduce renormalization freedom?
- If not, can we achieve SI by adding new ("induced") terms to $\mathcal{L}_{\mathsf{int}}$?

$$\rightsquigarrow d_{e_{i,j}^\kappa} T\left[\mathcal{L}_{\mathsf{int}}\cdots\right] \stackrel{?}{=} \partial_\mu T\left[\cdots Q^\mu_\kappa \cdots\right] - d_{e_{i,j}^\kappa}\mathcal{L}_{\mathsf{induced}}$$

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Perturbative SI strongly constrains the form of the interaction!

Strategy:

- Start with cubic ansatz of smallest possible UV-dimension for $\mathcal{L}_{\text{int}}$,
- The model will tell if higher order couplings are needed!

Example 1: QED to second order

- L-Q-pair is

$$L = A_\mu j^\mu, \quad Q^\mu_\kappa = w_\kappa j^\mu,$$

where $w_\kappa = I_e A_\kappa$ s.t. $d_{e^\kappa} A_\mu = \partial_\mu w_\kappa$ and $j^\mu = \overline{\psi}\gamma^\mu \psi$.

- Second order:

$$T[LL']|_{\text{tree}} = \langle\!\langle T\, A_\mu\, A'_\varrho \rangle\!\rangle\, j^\mu j'^\varrho + \text{fermionic contractions}$$

- fermionic contractions unproblematic but

$$d_{e^\kappa}\, \langle\!\langle T\, A_\mu\, A'_\varrho \rangle\!\rangle\, j^\mu j'^\varrho = \partial_\mu\, \langle\!\langle T\, w_\kappa\, A'_\varrho \rangle\!\rangle\ \, j^\mu j'^\varrho$$
$$+ c\, d_{e^\kappa}\, \{ I_e I'_{e'} ((ee')\eta^{\mu\varrho} - e^\varrho e'^\mu) \}\, \delta(x - x')\, j^\mu j'^\varrho.$$

- This is only a divergence if $c = 0$.

Example: Massless Yang-Mills theory

L-Q-pair

General ansatz:

$$L = g_{abc} A_{a\mu} A_{b\nu} (\partial^\mu A_c^\nu), \qquad g_{abc} \text{ arbitrary constants.}$$

- Split $g_{abc} = f_{abc} + d_{abc}$ s.t. $f_{abc} = -f_{acb}$, $d_{abc} = d_{acb}$,
- $d_{abc} A_{a\mu} A_{b\nu} (\partial^\mu A_c^\nu) = \frac{1}{2} d_{abc} \partial_\mu \left(A_{a\mu} A_{b\nu} A_c^\nu \right) \Rightarrow$ only f_{abc} survives,
- $d_{e^\kappa} L = \partial_\mu Q_\kappa^\mu \Leftrightarrow f_{abc}$ totally anti-symmetric,
- Thus: unique cubic Lagrangian of smallest possible UV-dimension

$$L = f_{abc} A_{a\mu} A_{b\nu} F_c^{\mu\nu}$$

$\rightsquigarrow$ shorter and simpler than in gauge theory: only physical degrees of freedom appear (no ghosts, no $\partial_\mu A_a^\mu$)

Second order tree graphs

$$
\begin{aligned}
T\,[LL']|_{\text{tree}} = \quad & f_{abc} f_{cxy} \Big\{ A_{a\mu} A_{b\nu} \, \langle\!\langle T \, F^{\mu\nu} \; F'^{\varrho\sigma} \rangle\!\rangle \, A'_{x\varrho} A'_{y\sigma} \\
& + 2 A_{a\mu} A_{b\nu} \, \langle\!\langle T \, F^{\mu\nu} \; A'^{\varrho} \rangle\!\rangle \, A'^{\sigma}_{x} F'_{y\varrho\sigma} \\
& + 2 A^{\nu}_{a} F_{b\mu\nu} \, \langle\!\langle T \, A^{\mu} \; F'^{\varrho\sigma} \rangle\!\rangle \, A'_{x\varrho} A'_{y\sigma} \\
& + 4 A^{\nu}_{a} F_{b\mu\nu} \, \langle\!\langle T \, A^{\mu} \; A'^{\varrho} \rangle\!\rangle \, A'^{\sigma}_{x} F'_{y\varrho\sigma} \Big\}
\end{aligned}
$$

- Since only $F^{\mu\nu}$ and A^{μ} appear, normalization is fixed by

$$
d_{e^\kappa} \, \langle\!\langle T \, A^{\mu} \; A'^{\varrho} \rangle\!\rangle = \underbrace{\partial^{\mu} \, \langle\!\langle T_0 \, w_\kappa \; A'^{\varrho} \rangle\!\rangle}_{\text{yields divergence} \Rightarrow \checkmark} + \underbrace{c \, d_{e^\kappa} \, \{ I_e I'_{e'} ((ee')\eta^{\mu\varrho} - e^{\varrho} e'^{\mu}) \} \, \delta(x - x').}_{\text{No divergence} \Rightarrow c=0}
$$

- $d_{e^\kappa_{i,j}} \, T[LL']|_{\text{tree}} = \partial_\mu T[\cdots Q^{\mu}_{\kappa} \cdots] - d_{e^\kappa_{i,j}} L_{\text{induced}}$
  $\Leftrightarrow$ f_{abc} satisfy Jacobi identity.

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& + 4 A^{\nu}_{a} F_{b\mu\nu} \langle\!\langle T\, A^{\mu}\; A'^{\varrho} \rangle\!\rangle\, A'^{\sigma}_{x} F'_{y\varrho\sigma} \Big\}
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Outlook: Self-interactions of helicity-two fields

Massless heli-2 field $h^{\mu\nu}(x,e) \rightsquigarrow \exists$ unique L-Q-pair (mod divergence)

$$\begin{aligned} L &= h^{\mu\nu}\left[(\partial_\mu h^{\alpha\beta})(\partial_\nu h_{\alpha\beta}) + 2(\partial^\alpha h_{\mu\beta})(\partial^\beta h_{\nu\alpha})\right] \\ &\overset{\text{div}}{\sim} \frac{1}{3}\left\{T^{(h)}_{\mu\nu} h^{\mu\nu} - F_{[\mu\kappa][\nu\lambda]} h^{\mu\nu} h^{\kappa\lambda}\right\}, \end{aligned}$$

where $T^{(h)}_{\mu\nu}$ is the string-local stress energy tensor.

Differences to helicity $s = 1$:

- Derivatives of the field of other form than $F_{[\mu\kappa][\nu\lambda]}$ enter L
  $\Rightarrow$ BDF construction is essential if renormalization induced from F
- Vanishing of "Ricci-trace" forces non-trivial choice of the propagator

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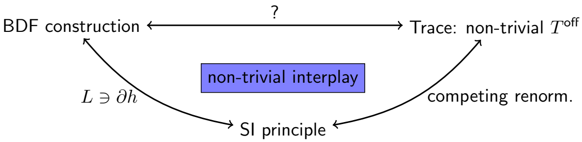

$\longrightarrow$ **future work**

Conclusions and general outlook

- Second order tree level: string-local version of QED and massless Yang-Mills theory agree with gauge theoretic versions
  $\rightsquigarrow$ expectation: agreement to all orders.
- Similar results in the massive case [Mund,GraciaBondía,Várilly – 2017].
- Implementation of a heli-2 self-interaction more involved.
- Compared to gauge theories: "string-local distributions" are more subtle, but the algebraic structure in the S-matrix is much simpler.
- String-local fields give a meaningful way to switch between field strengths and potentials.
- String-local loop graphs not yet considered!

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