String-localized fields of higher spin: massless limit and stress-energy tensor

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Abstract

"Lifting" the massless limit of Wigner **representations** of higher spin to the associated **local quantum fields**, encounters several obstructions due to the well-known conflicts between Hilbert space positivity, covariance and causality.

In a unified setting using "string-localization", these conflicts can be resolved, and details of the decoupling of the degrees of freedom can be studied.

Joint work with **Jens Mund, Bert Schroer**
(arXiv:1703.04407 and 04408)

Plan:

The lesson from “spin one”

Spin two

String-localized potentials

Higher spin

THE LESSON FROM “SPIN ONE”

The quantum Maxwell potential

"Canonical quantization" produces a conflict between Hilbert space positivity, covariance, and locality:

Quantum field A_μ such that $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ("curl")?

- Feynman gauge $\langle A_\mu A_\nu \rangle = \int d^4k\, \theta(k^0)\delta(k^2)[-\eta_{\mu\nu}]e^{-ikx}$: **indefinite**.
- ξ-gauges $[-\eta_{\mu\nu}\delta(k^2) + (\xi - 1)k_\mu k_\nu \delta'(k^2)]$: **indefinite**.
- Coulomb gauge $A_0 = 0$, $\langle A_i A_j \rangle = \left[\delta_{ij} - \frac{k_i k_j}{\vec{k}^2}\right]$: **not covariant, non-local**.

A positive, covariant, and local potential does not exist.

Only the field strength (= curl of either of the above) is positive:

$$\boxed{\langle F_{[\mu\nu]} F_{[\kappa\lambda]} \rangle = -\eta_{\mu\kappa} k_\nu k_\lambda + \eta_{\nu\kappa} k_\mu k_\lambda + \eta_{\mu\lambda} k_\nu k_\kappa - \eta_{\nu\lambda} k_\mu k_\kappa \ .}$$

Wigner quantization of free fields

- Wigner rep'ns of the Poincaré group = Hilbert space $\mathcal{H}_1$ of one-particle states (induced from unirep of stabilizer gp of k_0).

  massive: (half-)integer spin, $2s+1$ states (per momentum)
  massless: (half-)integer helicity, 1 state; or "infinite spin".

- Local free fields on Fock space $\mathcal{F}(\mathcal{H}_1)$ of the form

$$\Phi_i(x) = \int d\mu_m(k)\big[u_{i\alpha}(k)a_\alpha(k)e^{-ikx} + v\, a^*\, e^{+ikx}\big]$$

  transform covariantly iff $u_{i\alpha(k)}$ and $v_{i\alpha(k)}$ fulfil an intertwining condition between a matrix representation of the Lorentz group and the given unitary representation of the stabilizer group.

For integer spin, Wigner quantization yields

- $(m > 0, s)$: symmetric traceless rank s tensor fields $A_{\mu_1 \dots \mu_s}$ (generalizing the Proca field).
- $(m = 0, h = \pm s)$: rank $2s$ field strength tensors $F_{[\mu_1 \nu_1] \dots [\mu_s \nu_s]}$. (Single helicity fields are non-local).
- Intertwiners for massless potentials $A_{\mu_1 \dots \mu_s}$ do not exist.

Massless Wigner rep'ns are "contractions" of massive rep'ns (ie, the inducing massless stabilizer group $E(2)$ is a contraction of the massive $SO(3)$).
Apparently, this limit does not lift to the associated quantum fields.

$s = 1$: Massive case (Proca)

$$\boxed{\langle A_\mu A_\nu \rangle_m = -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m^2} \equiv -\pi_{\mu\nu}.}$$

- **Positive semi-definite**.
- UV-dim $= 2 \Rightarrow$ weak interaction **non-renormalizable**.
- Limit $m \to 0$ does not exist.

Define $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$, then

$$\boxed{\langle F_{[\mu\nu]} F_{[\kappa\lambda]} \rangle_m = -\pi_{\mu\kappa} k_\nu k_\lambda \pm \dots = -\eta_{\mu\kappa} k_\nu k_\lambda \pm \dots}$$

is exactly the same as for $m = 0$, except that $k^2 = m^2$.

- $F[m > 0]$ **converges to** $F[m = 0]$.

Moreover,

$$\partial^\nu F_{\mu\nu} = m^2 A_\mu$$

recovers the privileged (positive, covariant, local, conserved) potential from its field strength.

SPIN TWO

Why higher spin?

- Gravity (helicity 2)
- Why should Nature not use it?

"Spin 2" is similar:

$$\left\langle F_{[\mu_1\nu_1][\mu_2\nu_2]} F_{[\kappa_1\lambda_1][\kappa_2\lambda_2]} \right\rangle = \text{curls of} \left\langle A_{\mu_1\mu_2} A_{\kappa_1\kappa_2} \right\rangle$$

where

$$m = 0: \quad \boxed{\left\langle A_{\mu_1\mu_2} A_{\kappa_1\kappa_2} \right\rangle_0 = \tfrac{1}{2}(\eta_{\mu_1\kappa_1}\eta_{\mu_2\kappa_2} + \eta_{\mu_2\kappa_1}\eta_{\mu_1\kappa_2}) - \tfrac{1}{2}\eta_{\mu_1\mu_2}\eta_{\kappa_1\kappa_2},}$$

$$m > 0: \quad \boxed{\left\langle A_{\mu_1\mu_2} A_{\kappa_1\kappa_2} \right\rangle_m = \tfrac{1}{2}(\pi_{\mu_1\kappa_1}\pi_{\mu_2\kappa_2} + \pi_{\mu_2\kappa_1}\pi_{\mu_1\kappa_2}) - \tfrac{1}{3}\pi_{\mu_1\mu_2}\pi_{\kappa_1\kappa_2}.}$$

Both **field strengths are positive, covariant and local**, with $2s + 1 = 5$ resp. 2 one-particle states per momentum; but

- **Indefinite** Feynman gauge massless potentials do not exist on the Fock space, Coulomb gauge **non-covariant & non-local**.
- Massive potential is recovered from its field strength via $\partial^{\nu_1}\partial^{\nu_2} F_{[\mu_1\nu_1][\mu_2\nu_2]} = (m^2)^2 A_{\mu_1\mu_2}$. Positive, covariant, local, traceless and conserved. **No massless limit**.

. . . but different:

The "curls" do not see the difference between $\eta_{\mu\nu}$ and $\pi_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{m^2}$ in $\langle AA \rangle$;
– but they see the different coefficients $\boxed{-\frac{1}{2} \; vs \; -\frac{1}{3}}$ of the third term.

- Therefore also **the massive field strength does not converge to the massless field strength**.
- Even in lowest order (where non-renormalizability doesn't matter), or in indefinite gauges (where the massless potentials can be used), perturbative massive gravity does not converge to massless gravity (vanDam-Veltman–Zakharov 1970).

The UV dimension of the massive potential **increases with s.**

Weinberg-Witten (1980): **No local stress-energy tensor for** $m = 0$. (Field strengths involve too many derivatives!)

STRING-LOCALIZED POTENTIALS

Some answers in this talk:

Identification of potentials of any (integer) spin and any mass $m \geq 0$ that

- live on the respective Wigner Fock spaces,
- do admit a massless limit,
- have non-increasing UV dimension 1,
- quantify the DVZ discontinuity,
- admit massless stress-energy tensors.

The price: a weaker localization property

(. . . of the potentials, not of the particles!)

$$s = 1$$

For any mass $m \geq 0$, define op-val distributions in x and $e \in \mathbb{R}^4$

$$\boxed{A_\mu(x,e) := \int_{\mathbb{R}_+} d\lambda \, F_{\mu\nu}(x+\lambda e)e^\nu.}$$

Short hand:

$$A(e) = I_e F e = I_e curl(A) e.$$

These are

- potentials for their respective field strengths,
- defined on the respective Fock space, hence positive,
- regular at $m = 0$ (because F are),
- axial gauge potentials: $e^\mu A_\mu(e) = 0$,
- covariant: $U(\Lambda)A(x,e)U(\Lambda^*) = \Lambda^{-1}A(\Lambda x, \Lambda e)$,
- UV-tame: dimension 1,
- **"string-localized": the commutator vanishes when the two "strings" $x + \mathbb{R}_+ e$ and $x' + \mathbb{R}_+ e'$ are spacelike separated;**

Remark: Causality requires spacelike e, WLoG $e^2 = -1$.

Correlation functions

For any $m \geq 0$:

$$\boxed{\begin{aligned}\big\langle A_\mu(-e)A_\nu(e')\big\rangle_m &= -\eta_{\mu\nu} + \frac{k_\mu e_\nu}{(ek)_+} + \frac{e'_\mu k_\nu}{(e'k)_+} - \frac{(ee')\,k_\mu k_\nu}{(ek)_+(e'k)_+} \\ &\equiv -E(e,e')_{\mu\nu}.\end{aligned}}$$

The same formula for $m > 0$ and $m = 0$, except that $k^2 = m^2$.
Massless limit exists (as a limit of states on the Borchers algebra: the correlation functions define the fields).

The string-localized **massive potential converges to the massless potential** (not only the field strength).

$s = 2$ is different

$$\boxed{A_{\mu_1\mu_2}(x,e) := \int_{\mathbb{R}_+} d\lambda_1\, d\lambda_2\, F_{[\mu_1\nu_1][\mu_2\nu_2]}(x+\lambda_1 e+\lambda_2 e)e^{\nu_1}e^{\nu_2}}$$

$$A(e) = I_e I_e Fee = I_e I_e curl\, curl(A)ee.$$

Again, these are

- potentials for their respective field strengths,
- defined on the respective Fock space, hence positive,
- regular at $m = 0$ (because F are),
- covariant: $U(\Lambda)A(x,e)U(\Lambda^*) = (\Lambda\otimes\Lambda)^{-1}A(\Lambda x,\Lambda e)$
- UV-tame: non-increasing dimension $= 1$,
- string-localized.

But, unlike $s = 1$:

$$m > 0: \quad \boxed{\left\langle A_{\mu_1\mu_2} A_{\kappa_1\kappa_2} \right\rangle_m = \tfrac{1}{2}\left(E_{\mu_1\kappa_1} E_{\mu_2\kappa_2} + E_{\mu_2\kappa_1} E_{\mu_1\kappa_2}\right) - \tfrac{1}{3} E_{\mu_1\mu_2} E_{\kappa_1\kappa_2},}$$

$$m = 0: \quad \boxed{\left\langle A_{\mu_1\mu_2} A_{\kappa_1\kappa_2} \right\rangle_0 = \tfrac{1}{2}\left(E_{\mu_1\kappa_1} E_{\mu_2\kappa_2} + E_{\mu_2\kappa_1} E_{\mu_1\kappa_2}\right) - \tfrac{1}{2} E_{\mu_1\mu_2} E_{\kappa_1\kappa_2}.}$$

$A_{\mu_1\mu_2}(e)$ is regular in the massless limit, but **the limit is not the massless string-localized potential** (because of "$-\frac{1}{3}$ vs $-\frac{1}{2}$").
Instead:

$$\boxed{A^{(2)}_{\mu\nu}(e) := A_{\mu\nu}(e) + \tfrac{1}{2} E(e,e)_{\mu\nu}\, a(e)}$$

where $a(e) = -\eta^{\mu\nu} A_{\mu\nu}(e) = m^{-2}\partial^\mu\partial^\nu A_{\mu\nu}(e)$ and $E(e,e)_{\mu\nu} = \eta_{\mu\nu} + e_\mu I_e \partial_\nu + e_\nu I_e \partial_\mu + e^2 I_e I_e \partial_\mu \partial_\nu$ is the momentum space kernel of the 2-point function as an operator in x-space.

Proposition:

The (string-localized) field strengths of the potentials $A^{(2)}$ on the massive spin-2 Fock spaces converge to the massless field strength.

The DVZ discontinuity

"Massive linearized gravity" coupled to classical matter:

$$S_{\rm int}(e) = \int d^4x\, A^{\rm Proca}_{\mu\nu}(x)\, T^{\mu\nu}(x) = \int d^4x\, A_{\mu\nu}(x,e)\, T^{\mu\nu}(x).$$

Decompose

$$A_{\mu\nu}(x,e) = A^{(2)}_{\mu\nu}(x,e) - \frac{1}{2}\,\eta_{\mu\nu}\, a(x,e) + \text{derivatives},$$

where $\lim_{m\to 0} a(x,e) = \sqrt{2/3}\,\varphi(x)$ **decouples** from the massless helicity 2 potential $A^{(2)}(x,e)$. Thus,

$$\boxed{\lim_{m\to 0} S_{\rm int}(e) = \int d^4x\, A^{(2)}_{\mu\nu}(x,e)\, T^{\mu\nu}(x) - \sqrt{1/6}\int d^4x\, \varphi(x)\; T^{\mu}_{\mu}(x).}$$

The first term coincides with massless linearized gravity.

HIGHER SPIN TENSOR POTENTIALS

For any spin and $m > 0$, we define (short-hand)

$$A(e) = I_e^s F e^s = I_e^s curl^s(A) e^s,$$

which is regular at $m \to 0$.

- $A_{\mu_1 \dots \mu_s}(x, e)$ is neither traceless nor conserved.
- Its 2-point function is a polynomial in $E_{\mu\nu}(e, e')$.
- $A(x, e)$ differs from the singular privileged point-localized potential $A(x)$ by derivatives of its partial divergences $\partial^{\mu_{r+1}} \dots \partial^{\mu_s} A_{\mu_1 \dots \mu_s}(e)$:

$$A_{\mu_1 \dots \mu_s}(x) = \underbrace{(-1)^s \langle A_{\mu_1 \dots \mu_s} A^{\nu_1 \dots \nu_s} \rangle}_{\textit{differential operator Polynom}(\pi_\mu^\nu = \delta_\mu^\nu + m^{-2}\partial_\mu \partial^\nu)} A_{\nu_1 \dots \nu_s}(x, e).$$

- The latter subtract all the singularities of $A(x)$ as $m \to 0$, and are also expected to carry away the non-renormalizable UV singularities, when coupled to a conserved current.

Massless limit

For all $r \leq s$: let $a^{(r)}_{\mu_1\dots\mu_r}(e) := (-m)^{r-s} \cdot \partial^{\mu_{r+1}} \dots \partial^{\mu_s} A_{\mu_1\dots\mu_s}(e)$, and

$$\boxed{A^{(r)}_{\mu\dots\mu}(e) := \sum_{2k\leq r} \alpha^r_k \cdot (E_{\mu\mu}(e))^k a^{(r-2k)}_{\mu\dots\mu}(e)}$$

define string-localized tensor fields $A^{(r)}(e)$ of rank r on the same Fock space, regular at $m = 0$.

The coefficients α can be adjusted such that all $A^{(r)}(e)$ are traceless and decouple exactly at $m = 0$: $\boxed{\left\langle A^{(r)} A^{(r')} \right\rangle \sim \delta_{rr'} + O(m)}$.

Proposition:

The (string-localized) field strengths $F^{(r)}(e)$ of $A^{(r)}(e)$ converge to the point-localized massless field strengths of helicity $h = \pm r$. $A^{(0)}(e)$ converges to the e-independent massless scalar field.

Pauli-Lubanski limit

Proposition:

In the "Pauli-Lubanski limit"

$$\boxed{m \to 0,\ s \to \infty : \qquad s(s+1)m^2 = \kappa^2 = cst}$$

the "scalars" $A^{(0)}_{(m,s)}(e)$ converge to the massless infinite-spin field constructed by Mund-Schroer-Yngvason (2005).

More precisely (with R. Gonzo): the Wigner intertwiner of the limit violates the boundedness condition of MSY in the complex forward tube of e. This can be repaired by an additional operator

$$(1 + mI_e)^s\, A^{(0)}_{(m,s)}(e)$$

before taking the limit. The bound is secured by the resulting phase

$$e^{-i\kappa/(ke)_+}.$$

Stress-energy tensors

The stress-energy tensor at higher spin is not unique. We found

- The massive Hilbert SET (variation of the action by the metric) for $s = 2$ is different from the SET proposed by Fierz in 1939. Both produce the same generators of the translations (momentum operators), but different Lorentz generators.
- Only the Lorentz generators of the Hilbert SET implement the correct Lorentz transformations.
- We found a simpler "reduced" SET (quadratic in $A_{\mu\nu}(x)$, hence singular at $m \to 0$) that produces the same correct generators (not "derived from a Lagrangean").
- The reduced SET immediately generalizes to any $s > 2$.

Helicity decoupling

- We found yet another SET that is regular at $m \to 0$ and still produces the correct generators. It is quadratic in $a^{(r)}(x,e)$, hence string-localized.
- We found a massless SET that still produces the correct generators at $m = 0$. It is quadratic in $A^{(r)}(x,e)$, hence string-localized. Because $A^{(r)}$ mutually commute, this SET is actually a **direct sum of massless SETs for all helicities $h = \pm r$** present in the massless limit of the (m,s) representation:

Proposition:

$$T^{(r)}_{\rho\sigma}(x,e,e') = (-1)^r \Big[-\frac{1}{4} A^{(r)}_{\mu\times}(x,e)\, \overset{\leftrightarrow}{\partial}_\rho \overset{\leftrightarrow}{\partial}_\sigma\, A^{(r)\mu\times}(x,e') - \frac{r}{4}\, \partial^\mu \Big(A^{(r)}_{\rho\times}(x,e)\, \overset{\leftrightarrow}{\partial}_\sigma\, A^{(r)\times}_{\mu}(x,e') \begin{array}{l} +(e \leftrightarrow e') \\ +(\rho \leftrightarrow \sigma) \end{array} \Big) \Big]$$

$(\times \equiv \mu_2 \dots \mu_r)$.

INTERACTIONS

In progress: Causal perturbation theory

- Interaction Lagrangeans involving string-localized fields such that $S = \int dx\, L_{int}(e)$ is independent of e.
- Examples: QED, massive QED: $L_{int} = A_\mu(x, e) j^\mu(x)$. Mund, in preparation.
- Show that the Bogoliubov scattering matrix $S(g) = T \exp i \int g(x) L_{int}(x, e)$ is e-independent in the adiabatic limit.
- Construct renormalized string-localized fields connecting the vacuum to charged states.

- Explicit solution for $A_\mu(e)$ coupled to a classical conserved current: OK
- Similar for $A^{(2)}_{\mu\nu}(e)$ coupled to a classical matter stress-energy tensor?
- Standard model interactions: String-localized massive vector bosons must couple like gauge fields, their couplings to fermions must be chiral, and the presence of a Higgs field is required (in lowest orders)
  GraciaBondia-Mund-Varilly: arxiv:1702.03383,
  Mund-Schroer, in preparation.
- String-localized higher-spin SET coupled to (classical) gravity?
- Perturbative gravity?